IB Mathematics Exam Preparation
for Calculator Papers

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TEXAS INSTRUMENTS

Contents

Important notice regarding book materials

Introduction

This guide is designed to help students (and teachers) get the most out of using TI-Nspire™ technology as part of studying for the International Baccalaureate. Specifically it is designed to help students understand how the TI-Nspire can be used as an aid to answering exam questions.

The exemplar questions in this booklet are the author's own work, and are written in the style of IB questions.

The aim of this resource is that it would be useful during the course, but additionally it could also be used as revision material. Throughout there is as much emphasis on the mathematics as on the technology, as it is the author's belief that without the focus on the mathematics this guide would become little more than a button-pushing exercise.

Structure of the Guide

This guide is split into two main sections; the first a general overview of the TI-Nspire, and basic instructions on how to use it. This should be read before exploring the second part of the guide. The second section is a collection of IB style exam questions, with worked solutions using the TI-Nspire and comparing those solutions to the more traditional approaches. These questions are separated in the Mathematics HL, Mathematics SL and Mathematical Studies SL; although there is much overlap and you may find some of the questions from other levels of use and interest.

TI-Nspire Editions

This guide is based upon the TI-Nspire 2.1 Numeric handheld with the Touchpad. Much of it is still applicable to the TI-Nspire using the older ClickPad, however some of the key presses would be different.

Additionally it is important to remember that at present the IB does not allow students to use the CAS (Computer Algebra System) version of the handheld in the exam, so it is important to ensure that you have the correct version. The CAS versions have the letters CAS embossed on the reverse of the handheld.

Thanks

The author would like to thank the many people who made this booklet a possibility. Particular thanks must go to Marlene Torres-Skoumal who helped proof read the document and provided many useful insights into the markschemes. Additional thanks go to the useful advice given by those who attended my 2010 T[3] workshop in York where these activities were trialled.

TI-Nspire Layout

Start by spending a moment familiarising yourself with the layout of the keypad

TI-Nspire ™Touchpad
Use the center area as you would a laptop touchpad. Use the outer edges as right, left, up, and down arrows.

(esc) Removes menus or dialog boxes from the screen. Also stops a calculation in progress.

(🏠 on) Turns on the handheld. If the handheld is on, this key displays the home menu.

(≡📓) Opens the Scratchpad for performing quick calculations and graphing.

(doc ▾) Opens the Tools menu.

(tab) Moves to the next entry field.

(menu) Displays the application or context menu.

(ctrl) Provides access to the function or character shown above each key. Also enables shortcuts in combination with other keys.

(del) Deletes the previous character.

(⇧shift) Makes the next character typed upper-case.

(var) Displays stored variables.

Use these multi-purpose keys either alone or in combination with other keys, such as Ctrl.

(enter) Evaluates an expression, executes an instruction, or selects a menu item.

Overview

The TI-Nspire handheld is separated into a series of linked applications which have various uses in different situations. These are:

- **Calculator**

    o   This is the general all-purpose calculator environment and provides access to many useful features and functions via the menus

- **Graphs**

    o   This is the graphing environment which allows you to draw cartesian, parametric, polar, sequence and scatter plots. It also provides many useful tools to analyse and interact with these graphs.

- **Geometry**

    o   The Geometry environment is closely linked to the Graphing application, and also to performing geometric constructions. These interactive constructions can also be mixed with any of the graph types mentioned above.

- **Lists & Spreadsheets**

    o   This is a fully featured spreadsheet environment which allows you to work with data in a tabular form. It can also be used to capture data from a Graphs or Geometry page.

- **Data & Statistics**

    o   This is an environment for carrying out graphical statistical analysis, including dot plots, histograms, pie charts, box and whisker diagrams, scatter plots, etc.

- **Notes**

    o   The Notes application is designed to allow you to add explanatory notes to go alongside your work. It also provides access to the very powerful dynamic MathBox.

Changing the Settings

For various parts of the IB course you will need to change the settings on your calculator to move between angle measures, levels of accuracy and various other settings. These settings can be accessed in a variety of ways, but the simplest is probably to go to the Home page (⌂on) then go to 'Settings & Status' at the bottom of the page (⑤) then choose 'Settings' (②) then select 'General' (①). This will present you with the following options:
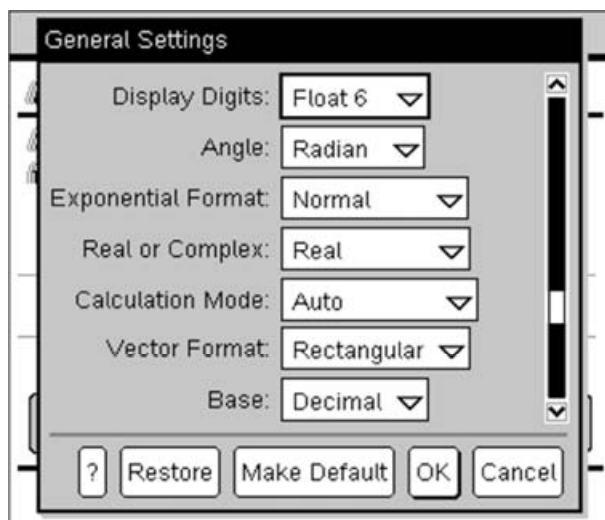
The 'Display Digits' option at the top determines the accuracy of the answers shown. To change this to 3 significant figures, change this to 'Float 3'.

The 'Angle' option allows you to move between degrees and radians.

The other thing that is of interest to those studying the HL course is the ability to change whether the TI-Nspire will display complex solutions. In this you have the choice of Real, Rectangular ($a + ib$) or Polar ($e^{i\theta}$) solutions.

These settings affect all calculations on the TI-Nspire, except those on a Graphs or Geometry page, or those carried out on the ScratchPad.

The settings for the ScratchPad are always set to be the default settings (i.e. the settings used when a new document is created). To change this, select 'Make Default' on the above settings menu instead of choosing OK.

To change the Settings for a Graphs or Geometry page, again go to the Home page ((⌂on)) then go to 'Settings & Status' at the bottom of the page ((5)) then choose 'Settings' ((2)) this time select 'Graphs & Geometry' ((2)).
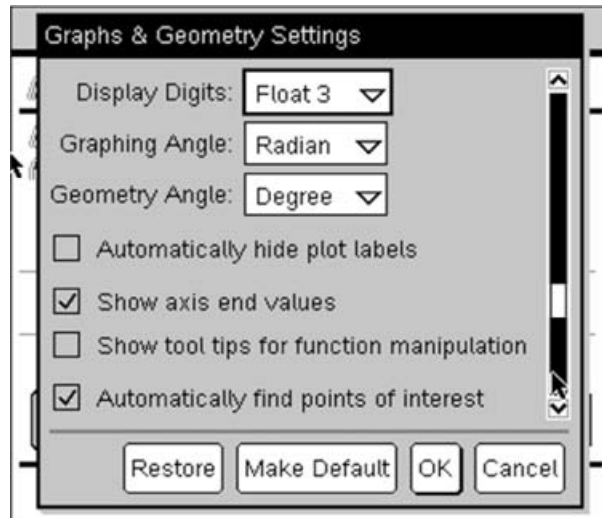
Notice here that the default settings are for graphs to be calculated in radians, but for angles to be measured in degrees.

Hints and Tips

The following is a collection of useful hints and tips (there are many more mentioned throughout this document). If you want a full step by step introductory guide to the TI-Nspire I would recommend looking at the 'First Steps with TI-Nspire' booklet[1].

Moving Between Pages – To move between pages press (ctrl) and then left or right to move to the previous or next page.

Contextual Menu – Many things in the TI-Nspire have additional properties and options. These can often be accessed by right-clicking, which is done on the handheld by pressing (ctrl)(menu).

Approximate Solutions – If you want to calculate a decimal approximate solution to a calculation rather than returning a fractional answer, press (ctrl)(enter).

ScratchPad – The ScratchPad is a simple calculator/graphing page that can be opened over the top of whatever you are doing by pressing (=🗐). Pressing (=🗐) again will swap between calculator or graphing ScratchPad. To exit the ScratchPad press (esc).

Menus – The TI-Nspire is heavily menu-driven so to explore the wide range of things that are possible press the (menu) key and look at the options that are available in the application you are working in.

Templates – There are lots of useful templates which make it easier to work with the Mathematics. To access the templates menu press (≡{‡)(del).
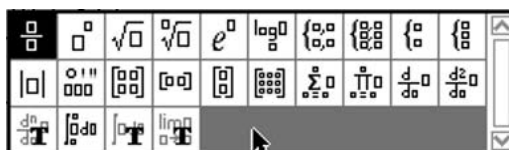
Trigonometric Functions – Unlike most traditional calculators you will not find sin, cos and tan keys on the TI-Nspire. These are instead replaced with a single (trig) key which gives you access to all six of the trigonometric ratios and their inverses.

sin	cos	tan	csc	sec	cot
sin⁻¹	cos⁻¹	tan⁻¹	csc⁻¹	sec⁻¹	cot⁻¹

Useful Symbols – Useful symbols such as π, i, etc are available by pressing the (π▸) key on the bottom left of the keyboard. The **!**symbol for factorials can be found by pressing (?!▸). To access the complete symbol bank press (ctrl)(😐).

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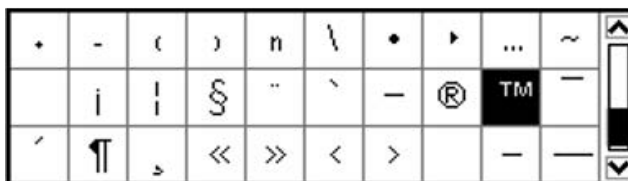
[1] 'First Steps with TI-Nspire' can be downloaded from: **http://resource.nspiringlearning.org.uk/downloads/**

Using the History – A very useful feature of the calculator page is the ability to reuse calculations and answers. To do this simply scroll up through the history by pressing up on the TouchPad, then when you reach a calculation or answer you want to reuse press (enter) to copy it down to the current line.

Grabbing and Dragging – When you are working on a Graphs or Geometry page, it is possible to manipulate many of the objects (includes axes, and many simple graphs) by picking up the object. You can grab an object by moving your cursor over the top of the object and either pressing and holding the click button (⌖) at the centre of the TouchPad or by pressing (ctrl) then clicking ⌖. To drop the object simply click (⌖) again.

Graph Entry Line – You can show or hide the graph entry line by using the shortcut (ctrl)G.

Adjusting Accuracy – On a Graphs or Geometry page it is possible to increase or decrease the displayed accuracy of any numbers showing by hovering your cursor over the number and pressing ⊕ or ⊖.

Variables – When exploring certain areas of mathematics it can be useful to store some of the numbers as variables. On a Graphs or Geometry page you can do this by selecting the value you want to store, then pressing (ctrl)(var), then typing a variable name. On the calculator page if you want to store the previous answer, or a number you have just typed in, press (ctrl)(var) and enter your variable name.

Functions – It is often useful when working with a function to define it so that you can reuse it throughout your work. For example to define the function $f(x) = x^2 + 5x + 6$ on the calculator page we would press F(X)(ctrl)(:=)X(x²)⊕5X⊕6. The **:=** that is entered by pressing (ctrl)(:=) means 'define'. Once you have done this you can do things like find f(10), by typing F(1)(0)(enter).

Splitting the Screen – In any application you can split the screen. To put another application side by side on the same screen, press (doc▾)(5)(2).

Typing Functions – Rather than accessing functions from the menus or catalogue (📖), you can manually type them using letter keys on the keypad. For example to find $^{10}C_5$, you can just press N C R (1)(0)(,)(5)(enter).

Press to Test Mode

During all IB exams the TI-Nspire must be placed into Press-to-Test mode with 'limited geometry features', and 'drag and grab' enabled (i.e. unticked). Press-to-Test mode replaces the traditional reset. In this mode you lose access to all your files, and the following menus (and related submenus) are disabled when on a Graphs or Geometry page:

- ***Menu 8 – Measurement***

- ***Menu A – Constructions***

- ***Menu B – Transformations***

With those exceptions the TI-Nspire behaves exactly the same in Press-to-Test mode. The advantage of Press-to-Test over the traditional reset approach is that when you exit Press-to-Test after an exam, the TI-Nspire will return to its previous state with your files still intact.

To put your TI-Nspire into Press-to-Test mode, first switch off the handheld. Hold down (esc) and press (⌂ on). This will present you with the following screen:
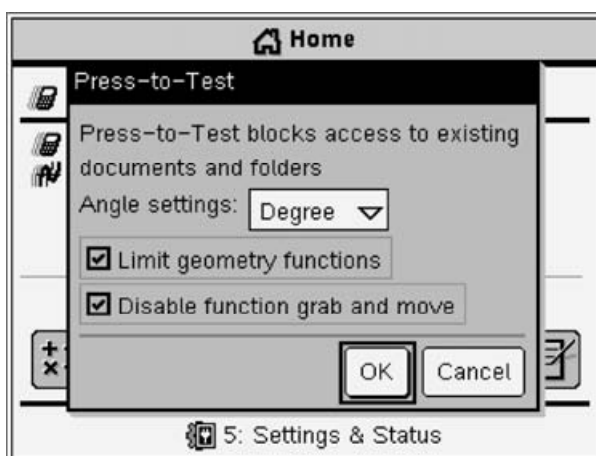
You should untick the 'Disable function grab and move' option, and set the Angle settings to whatever is most appropriate for you (for Mathematics HL and SL this is probably Radians, for Mathematical Studies this will be Degrees).
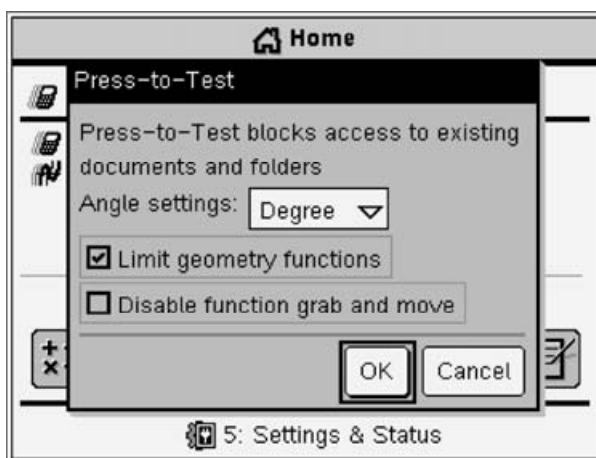
The simplest way to exit Press-to-Test mode is to connect your handheld with the link cable to another TI-Nspire (which can also be in Press-to-Test mode) and press (doc▾)(9) which will reboot the handheld.

Exam Questions

Whilst some of the questions involve topics which are specific to a particular IB course, they all involve techniques which may be relevant or interesting whichever course you are studying.

The following gives some indication of which questions are relevant to each level:

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[2] Whilst this example is set in a context not relevant to the Mathematical Studies course, the skills and techniques included are still very relevant to this course.

Finance:

> **Question:**
> £750 is invested in a high interest savings account, with an interest rate that is accrued on a monthly basis. After 4 years the account is now worth £1100. Find the annual interest rate of the account.
>
> [3 Marks]

The TI-Nspire has a built in Finance Wizard for dealing with this sort of Time, Value & Money (TVM) problem. The key to solving questions like this using the TI-Nspire is to be clear about what information we know and what we want to find out.

To use the Finance Solver we must first create a Calculator page (ctrl doc then select 'Add Calculator'). The Finance Solver can then be called by pressing menu 8 1. This will present you with the following screen:
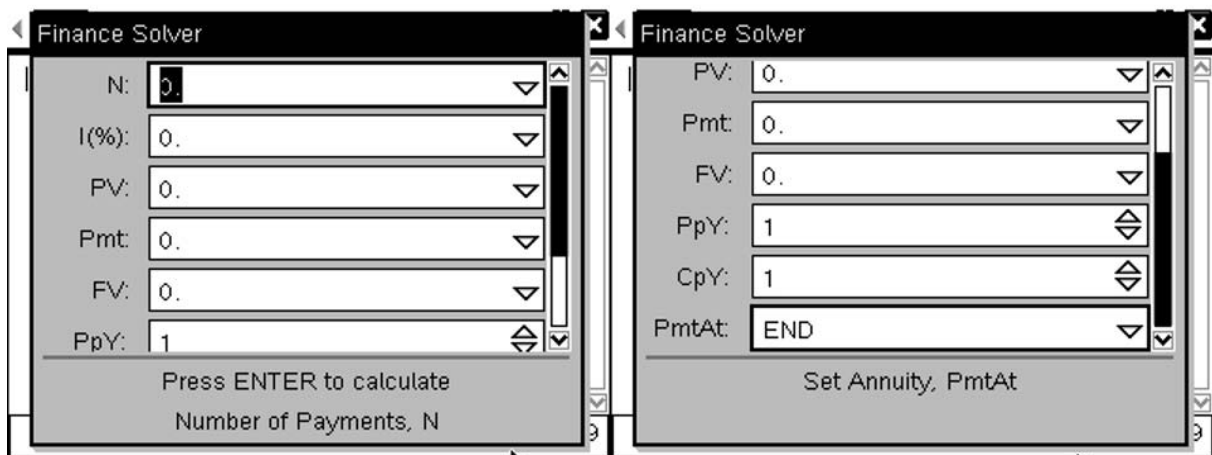
This has a number of fields which can be completed. However to use the solver you will need to know what they mean! Thankfully there is a reminder of what each means displayed at the bottom of the window as you move between the fields. i.e. "Number of Payments, N".

The various fields are:

N – This is the number of periods of time involved in the problem. For our example this is 4 years, but expressed in months (as we are calculating interest on a monthly basis), so N=4x12=48

I(%) – This is the interest rate of the loan or investment. In this case this is the information we want to know.

PV – This is the Present Value of the investment or loan. **It is important to remember that this value is negative for investments.** So for this example PV=-750

Pmt – This is the amount being regularly paid in to the account. Again this is negative for investments. For our question we are investing the money but not adding to it so Pmt=0

FV – This is the Future Value of the investment or loan. In our case we want the investment to be worth £1100 after the 4 years so we set FV=1100 (*notice this is positive*)

PpY – This is the number of Payments (of the Pmt amount) per year. For example if you were paying in monthly you put this as 12, quarterly as 4 and annually as 1. In our case as we are not making payments

this value is unimportant and can be left as 1. This is also used to adjust the returned Interest Rate, as if N is months and PpY=1, then the interest rate given is the monthly one. To adjust it to be the annual one, PpY must be 12.

CpY – This is the number of compound interest periods per year. If your interest is compounded annually you would leave this as 1, and N would be the number of years. If it was calculated daily you would put this as 365, and N would be 365 times the number of years. For our example, we are calculating the interest on a monthly basis so CpY=12 and our N value is 12 times the number of years.

PmtAt – This option allows you to change whether the payment is made at the beginning of the period or at the end. For our example as there is no payment this can be ignored.

To calculate the interest rate as desired we must enter the values from the question as follows:

- First enter **N**, as ④⊗①② (or just enter 48 directly), then press the (tab) key to move to the next field.

- The next field is I(%) which is what we want to calculate so we will skip over this entry for now by pressing (tab) again.

- Next is PV, which needs to be entered by pressing (–)⑦⑤⓪, then pressing (tab) to move to the next field

- Next is Pmt, which we are not using so press (tab) to move to the next field

- Next is FV, which for our question is entered as ①①⓪⓪. Then press (tab) to move to the next field

- Next is PpY, which needs to be changed to 12, so that the interest rate returned is per annum rather than per month, by pressing ①②. Then press (tab) to move to the next field

- Next is CpY, which needs to be changed to 12, by pressing ①②. Then press (tab) to move to the next field

- Finally we are at PmtAt, which can be ignored for this example.

- We want to calculate the interest rate, so we press (tab)(tab) to move back round to the I(%) field. To calculate its value simply press (enter).
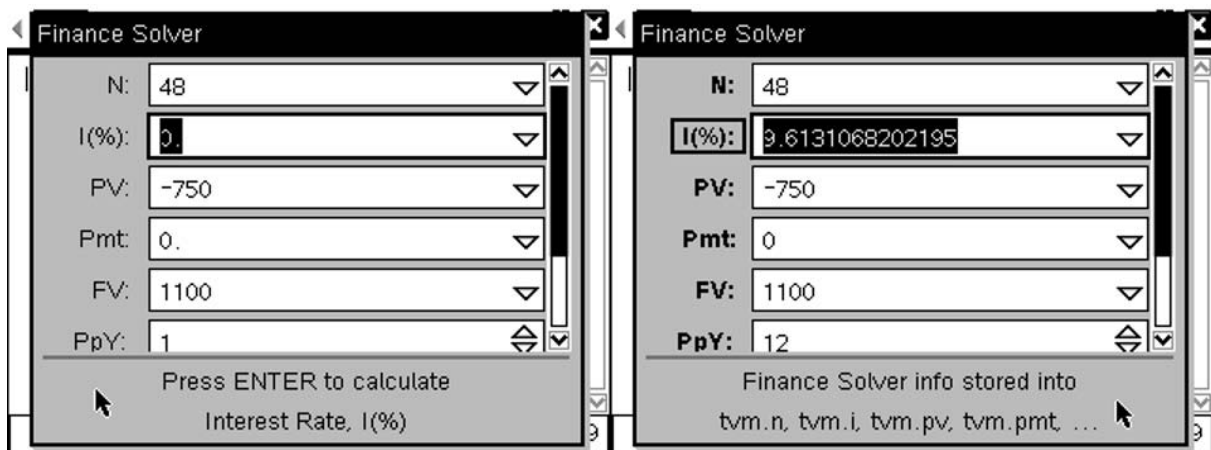
Alternatively, we could have set N to be 4, and left PpY as 1, this would have given the same solution:
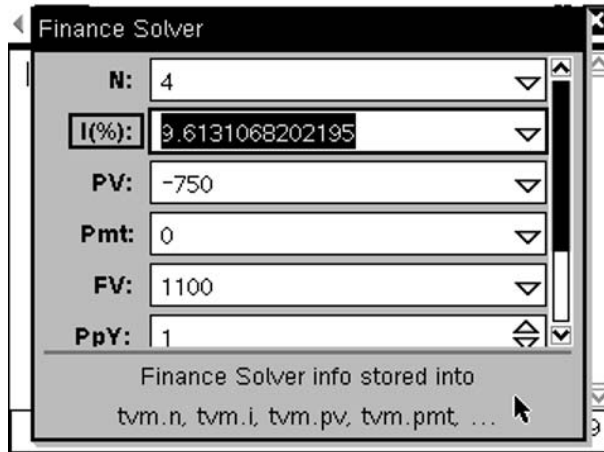
If we had accidentally forgotten to enter the Present Value as a negative number, we would have obtained the following error message.
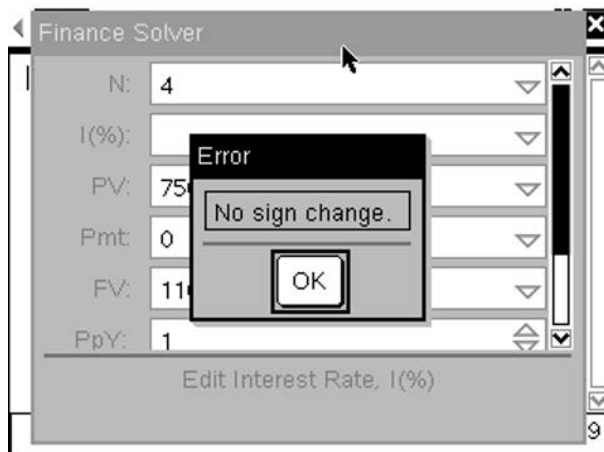
The more traditional approach to this problem involves the use of nth roots, to solve the compound interest equation:

$$I = C\left(1 + \frac{r}{100k}\right)kn - C$$

In this case I=£350 (1100-750), C=£750, n=4 (number of years), k=12 (number of compounding periods per year) and r is the monthly interest rate:

$$350 = 750\left(1 + \frac{r}{100 \times 12}\right)^{4\times12} - 750$$

$$1100 = 750\left(1 + \frac{r}{100 \times 12}\right)^{4\times12}$$

$$\frac{1100}{750} = \left(1 + \frac{r}{100 \times 12}\right)^{48}$$

$$\sqrt[48]{\frac{1100}{750}} = \left(1 + \frac{r}{100 \times 12}\right)$$

$$\sqrt[48]{\frac{1100}{750}} - 1 = \frac{r}{100 \times 12}$$

$$12 \times 100\left(\sqrt[48]{\frac{1100}{750}} - 1\right) = r$$

$$r = 9.61311\%$$
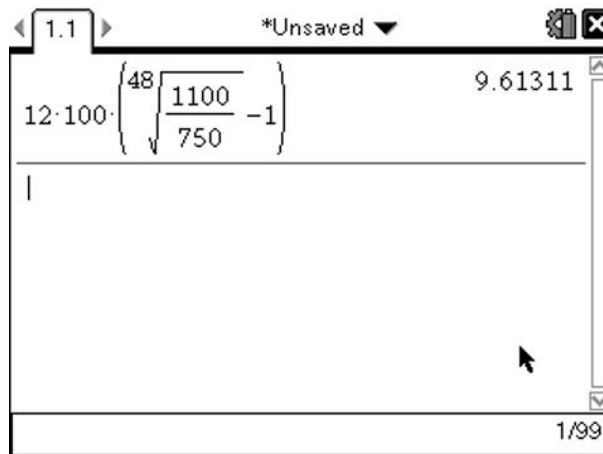
So both methods get us the same answer. One of the major advantages of the TI-Nspire based Solver approach is the ability to explore options by changing any one of the values and exploring its effect upon the problem without having to carry out all the calculations again. However like any heavily technological approach it is essential that you have some intuitive feel for the problem otherwise it is easy to fail to spot a nonsensical answer.

Marksheme:

$$I = C\left(1 + \frac{r}{100k}\right)^{kn} - C$$

$$350 = 750\left(1 + \frac{r}{100 \times 12}\right)^{4 \times 12} - 750 \qquad \text{M1}$$

$$\frac{1100}{750} = \left(1 + \frac{r}{100 \times 12}\right)^{48}$$

$$r = 12 \times 100\left(\sqrt[48]{\frac{1100}{750}} - 1\right) \qquad \text{(A1)}$$

$$r = 9.61\% (3sf) \qquad \text{A1}$$

Note that as only the A1 is unbracketed at the end, this implies you would obtain full marks for a correct numerical solution given to 3 significant figures irrelevant of the method used, or working included.

Small Data Sets:

> **Question:**
> For the following data set: {7,2,9,3,2,9,5,1,1,7,9,6}
> Find the:
>   i)   Mean
>   ii) Median
>   iii) Mode
>
> [3 Marks]

For a small set of data like this we could carry out these calculations by hand or using the TI-Nspire. In an exam situation, if you have the time, it is often wise to do it both ways to be sure you haven't made any errors.
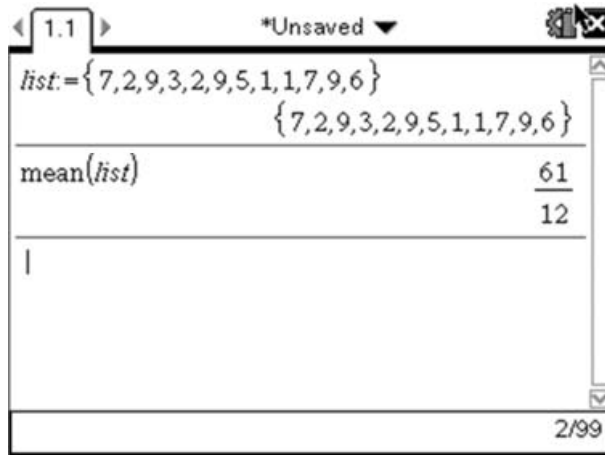
The first part of the question asks us to find the mean value for this data set. To do this by hand we add together all the values in the data set and divide by the number of values. So for our example we would do:

$$\bar{x} = \frac{7+2+9+3+2+9+5+1+1+7+9+6}{12} = \frac{61}{12} = 5\frac{1}{12}$$

The TI-Nspire has a mean function that does this calculation. We first need to enter our data set. We do this by creating a Calculator page (⌃ docᵥ) then select 'Add Calculator'). Once we have a Calculator page we can enter our data as a list by putting it inside curly {} brackets. We will call our data set, 'list' (but we could call it anything!). To enter our data we press L I S T then press ⌃ ᵢ≔ᵢ which enters ':='. This means we want to make a list equal to whatever follows. Now press ⌃ ⑴ to insert a pair of curly brackets and enter our data using the ⦿ to separate the values. This should give the following:
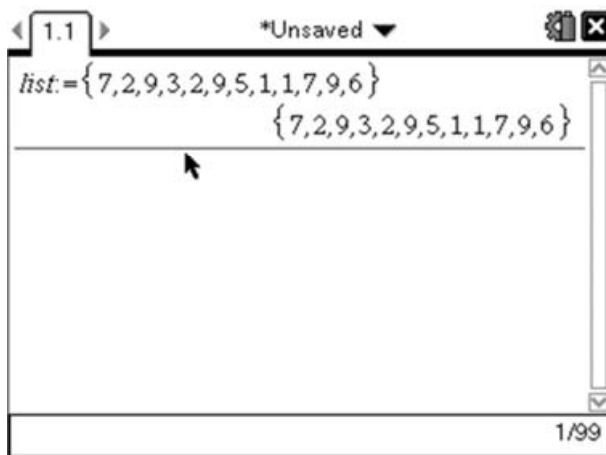
Now to find the mean of this data set we can either just type mean(list) (i.e. M E A N ⑴ L I S T ⑴), or use the menus by pressing ⟨menu⟩ ⑥ ③ ③ to insert the mean function, then type L I S T followed by ⟨enter⟩:

This value corresponds to the answer we found doing the question manually.

The second part of the question asks us to find the median value. Doing this by hand we would sort the list into numerical order and then find the middle value. This gives us the following:

$$7,2,9,3,2,9,5,1,1,7,9,6$$

$$1,1,2,2,3,5,6,7,7,9,9,9$$

The median value is the $\frac{n+1}{2}^{th}$ value in the list, which for this data set is the 6.5[th] term. This means we need to find the middle of the 6th and 7th terms:

$$median = \frac{5+6}{2} = \frac{11}{2} = 5.5$$

Using the TI-Nspire we can find the median in a very similar way to the method we used to find the mean. As the list has already been entered, we simply need to use the median function to find the median value for the data set. Like the mean function we can do this by typing $\boxed{M}\boxed{E}\boxed{D}\boxed{I}\boxed{A}\boxed{N}\boxed{(}\boxed{L}\boxed{I}\boxed{S}\boxed{T}\boxed{)}$ or by pressing (menu)(6)(3)(4) to insert the median function, then entering the list by pressing (var), then selecting list. Finally press (enter) to run the command:
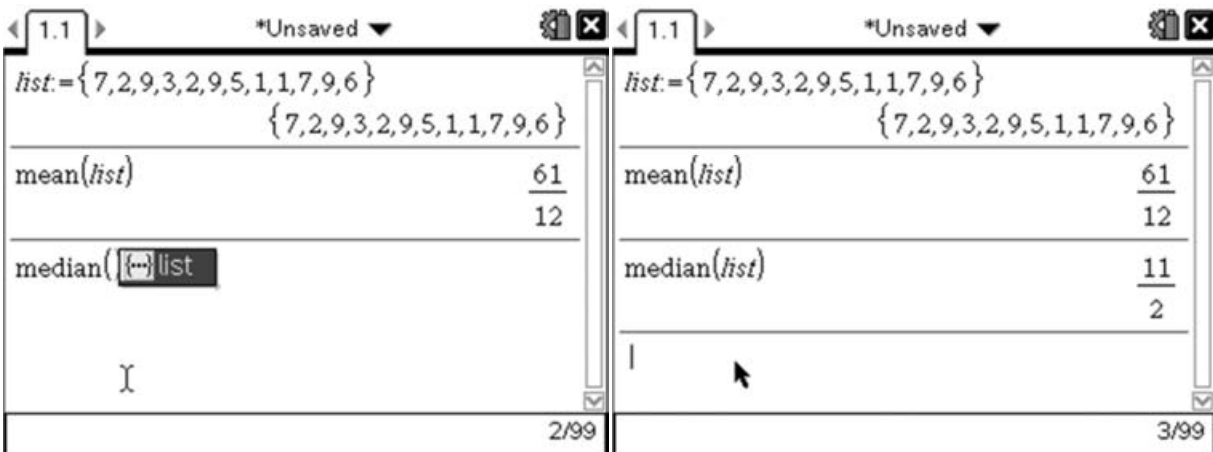

Again this result agrees with the value we found manually.

The third part of the question asks us to find the modal value. To do this we have to look through the list and find the value that occurs most often. Given that we have already sorted the list for the manual method we can now look at this list to identify the mode:

$$1,1,2,2,3,5,6,7,7,9,9,9$$

It is clear that there are more 9's than any other value so 9 is the mode.

On the TI-Nspire there isn't a built-in mode function, so we have to approach this part slightly differently. One option is to get the TI-Npsire to sort the list then manually inspect the list as we did above. To do this use the SortA function by pressing (menu)(6)(4)(1) followed by the name of our list, which we can enter by either typing its name or pressing (var) and selecting the appropriate variable. This will re-sort this list but not display the sorted list. To do this we simply type the name of the list ((L)(I)(S)(T)) followed by (enter).
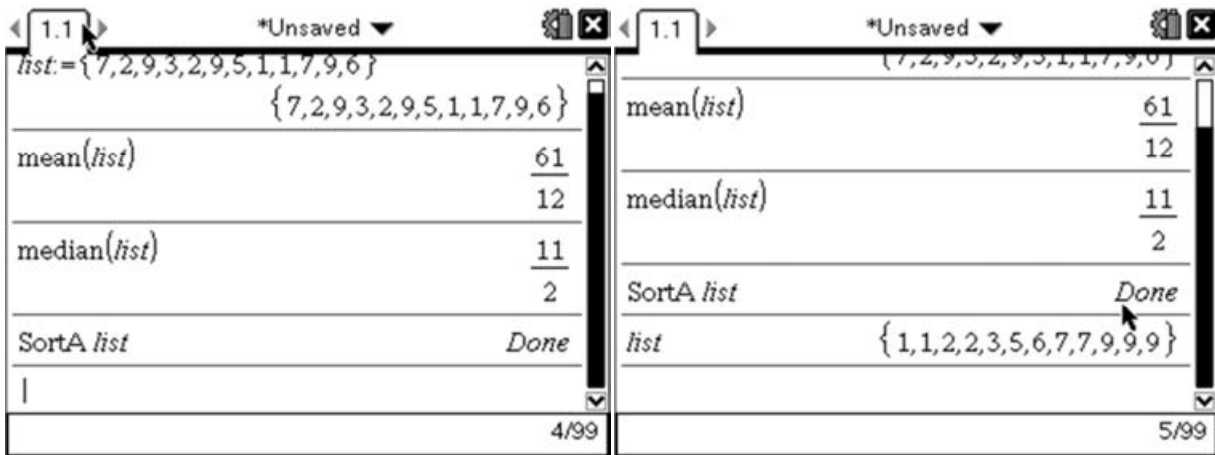
Then using the same approach as we used above we can see that there are more 9's and therefore 9 is the mode.

Alternatively we could draw a dot plot of the list, as this would enable us to quickly identify the mode, as it would be the tallest 'bar'. To do this, add a Data & Statistics page ((ctrl)(doc▾)) and select 'Add Data & Statistics'):
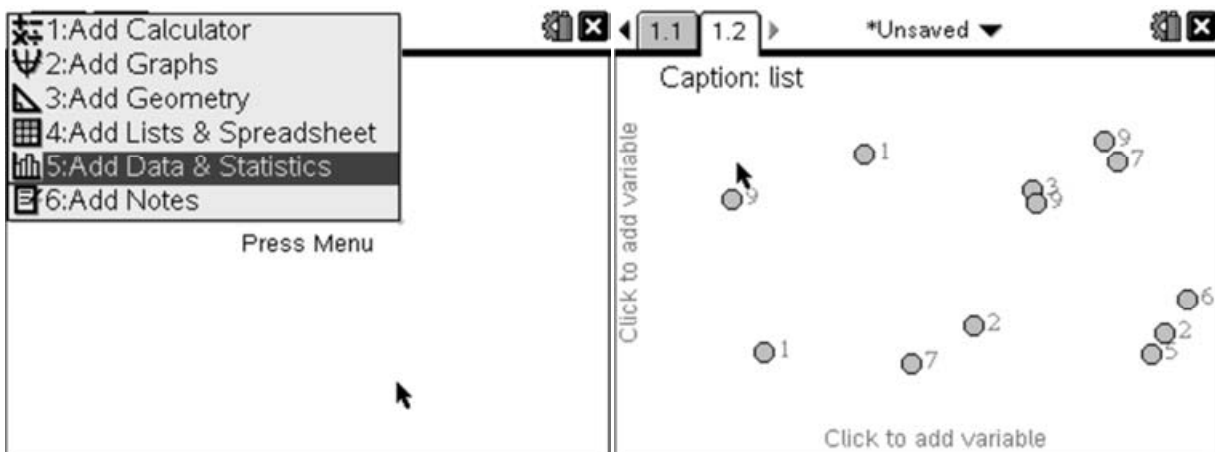
Now we need to add our list as the x-variable. To do this, click on the section at the bottom labelled 'Click to add variable' then select 'list':
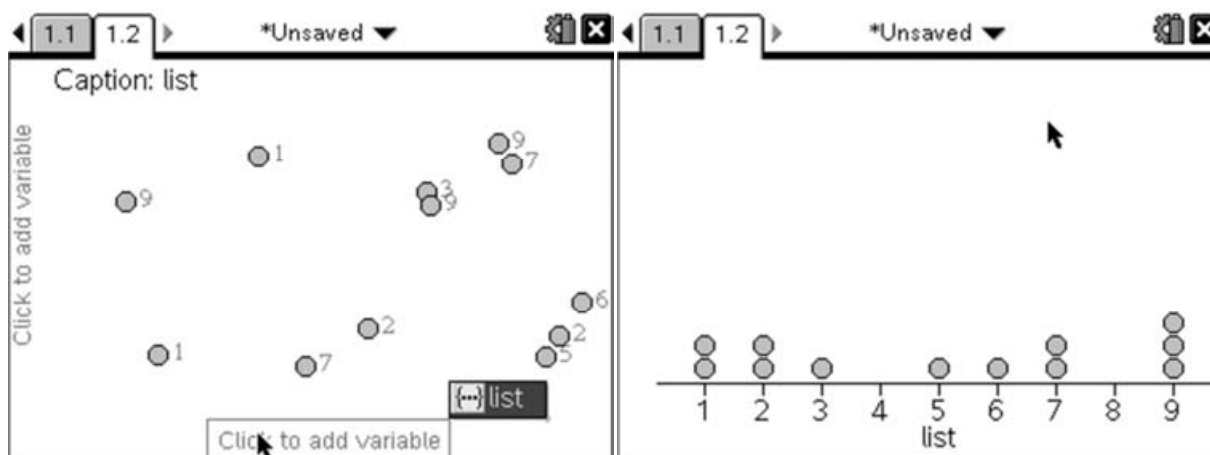
This gives us a dot plot for the data set, and the tallest 'bar' here represents the most frequent value, which corresponds to the mode. So we can quickly see that the mode is again 9.

Whilst it is desirable to carry out these types of questions manually, it is a very important to become proficient in using the TI-Nspire.

Markscheme:

Mean $= 5\frac{1}{12}$ A1

Even number of terms, so mean of middle two terms $\frac{5+6}{2}$

Median $= 5\frac{1}{2}$ A1

Mode = 9   A1

For a question like this you would expect to earn full marks for correct numerical answers (given to appropriate accuracy). Where there are intermediate steps, like finding the median, there is the potential to gain method marks if a correct method was implied with an incorrect answer.

In the shoe column, enter the shoe sizes as displayed in the table in the question, followed by the related frequencies in the freq column:

Now that we have the data in the calculator we can carry out some statistical analysis upon it. For example, we could draw a bar chart by pressing (menu)(3)(5) which will display the dialogue box where you can choose 'data list' and 'frequency list'. You could also then change this bar chart to be a box plot by pressing (menu)(1)(2).

However, in this question we are asked to carry out a more numerical analysis of this data. Again, the TI-Nspire has a built-in routine for carrying out basic statistical analysis. We can access this by pressing (menu)(4)(1)(1)which will start the 1 variable statistics calculation. There is a temptation here to think that we have two variables as we have two lists, but the only variable is shoe-size; the frequency column is just a shorthand way of recording this data.
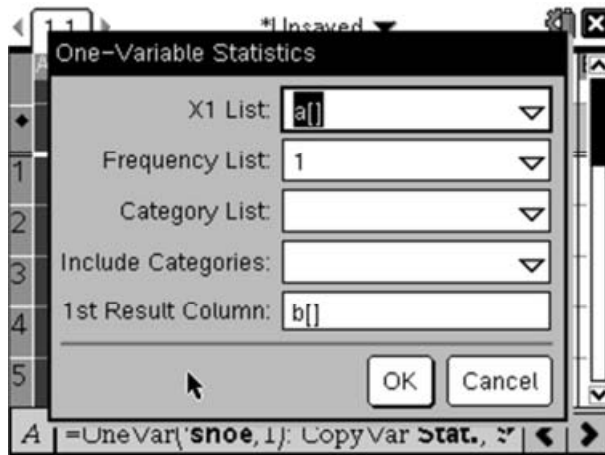

This presents you with a dialogue box asking for the number of lists; this can be slightly confusing as it really means data-sets. We only have one data-set despite having two lists (one being the frequency) so this should be left as 1. So press (enter) to move on to the next part of the process:

This asks us to say what our list of x-values is, and give a frequency list (if appropriate). If our list was a single list of everyone's shoe size i.e. 4,4,4,5,5,6,6,… etc then we would leave the 1 in the Frequency List option. However for this example the X1 List should say 'shoe', so press down to display the drop-down list and choose 'shoe':
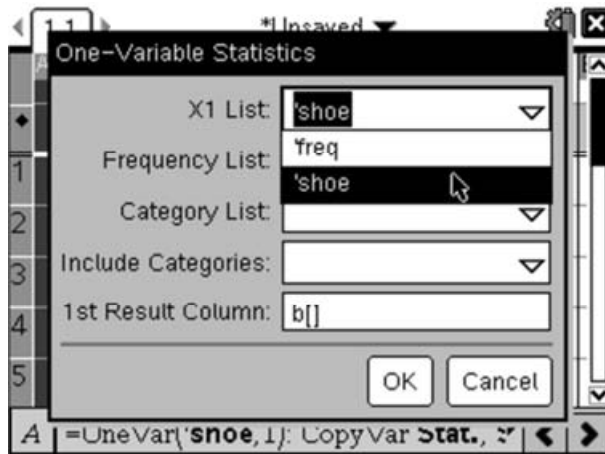
Then press (tab) to move to the Frequency List and press down to select 'freq'. We do not need to adjust the remainder of the settings so we press (enter) to complete the wizard.

This will add various calculated values to columns C and D as follows:
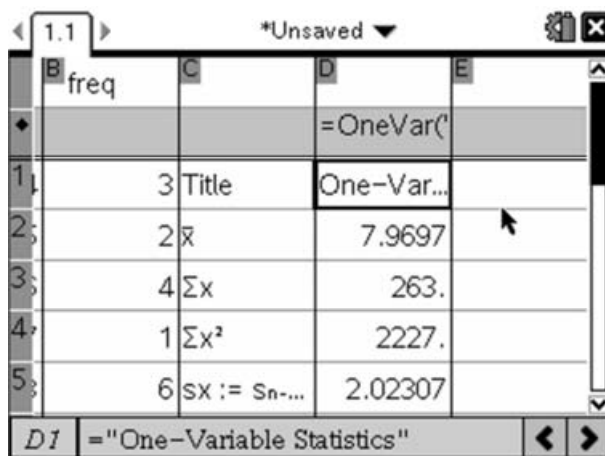
The first entry in this list ($\bar{x}$) is the mean, so the answer to the first part of the question is that the mean is 7.697 (to 3dp). The next two values give you the sum of the x values (i.e. sum of shoe-size x freq), and sum of the x^2 values (i.e. sum of shoe-size^2 x freq).

The next two give you the standard deviation; sx is the standard deviation if the data represents a sample, and σx gives the unbiased estimate of the population standard deviation. The next value (n) tells you how many data values there are (i.e. sum of the frequency).

The next five numbers represent what is sometimes called the 'Five Number Summary' and gives the data necessary to draw a box plot for the data. MinX is the minimum value, Q_1X is the lower quartile, MedianX is the median value, Q_3X gives the upper quartile and MaxX is the maximum value.

So we can now finish the question as the second part asked for the Median, which we now know is 9. The final part asks for the Inter-Quartile Range. This is the difference between the upper quartile and the lower quartile, so for this data set we have $IQR = Q_3 - Q_1 = 9 - 6 = 3$

Markscheme:

Mean = $\dfrac{4\times3+5\times2+6\times4+7\times1+8\times6+9\times10+10\times5+11\times2}{3+2+4+1+6+10+5+2}$ (M1)

Mean = $\dfrac{263}{33} = 7.97 \ (3sf)$ A1

Median   = Middle number of 33

       = 17$^{\text{th}}$ Number

       = 9   A1

IQR   = $Q_3 - Q_1$ (M1)

       = $9 - 6$

       = 3   A1

In this question as all the method marks are bracketed full marks would be awarded for a correct numerical answer irrespective of working shown, as the method would be implied. However incorrect numerical solutions with a correct method could still gain half the marks.

Graphs of Functions:

> **Question:**
> Sketch the following functions:
>   i) $f(x) = \sin x$
>   ii) $g(x) = 2e^{-x}$
>   whose domains are restricted to $-1 < x \le 2$
>
> Hence solve the equation $f(x) = g(x)$
>
> [4 Marks]

The first thing to notice here is that the domains of these functions are restricted to values of x between -1 and 2. We may be able to sketch these functions without the aid of a GDC, however it is always sensible to check our sketch where possible.

To draw these functions on the TI-Nspire we must first insert a graphing page (ctrl doc▾ then select Add Graphs) and enter our two equations as f1 and f2. To enter the first function press (trig) to bring up the trigonometric functions and select the *sin* function, then press the ⊠ key followed by (enter) to draw the graph.

To enter the second function we need to bring the function entry line back up. We can do this in two ways; either by pressing (tab) (if the cursor is not selecting anything) or by pressing ctrl G. Once we have the new entry line up we can enter the second function by pressing ②(e^x)(-)⊠ followed by (enter) to draw the function. This will give us the following graph:
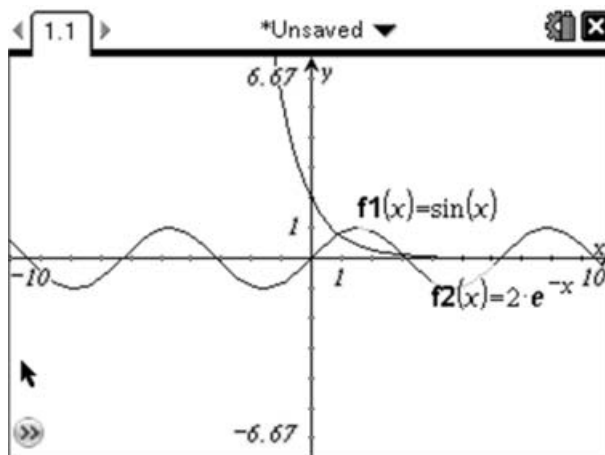

This graph gives us a good overview of the general shape of the two functions, and indicates that the functions cross in two places somewhere near 1 and somewhere near 3. However we must not forget that the domain of this function is $-1 < x \le 2$, so we should adjust the window settings to only display the relevant parts of the graph.

There are several ways we can adjust the window settings. One option is to grab (✦) and hold or ctrl✦) on to one of the tick marks on an axis, then drag it towards the centre to zoom in or towards the edge to zoom out. Holding down (⇧shift) whilst doing this will zoom in only one axis. However in this case, as we know exactly what size we need, we will use the 'zoom and windows' settings available from the Menu. Pressing (menu)④ gives us the following dialogue box:
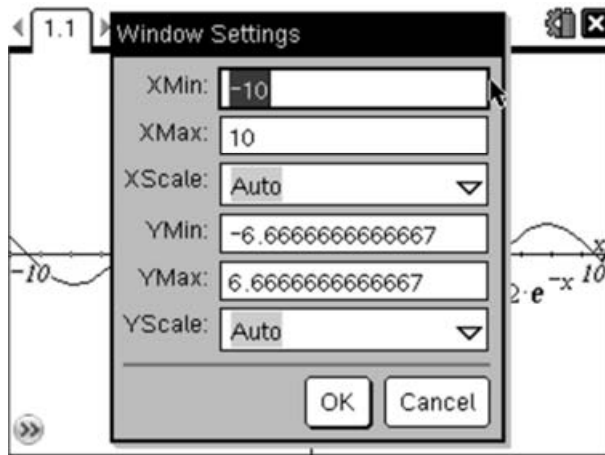
We can adjust the XMin and XMax values here to be -1 and 2 respectively. Simply type over the current value and use (tab) to move between the options. Finally press (enter) to close the dialogue window. This gives us:
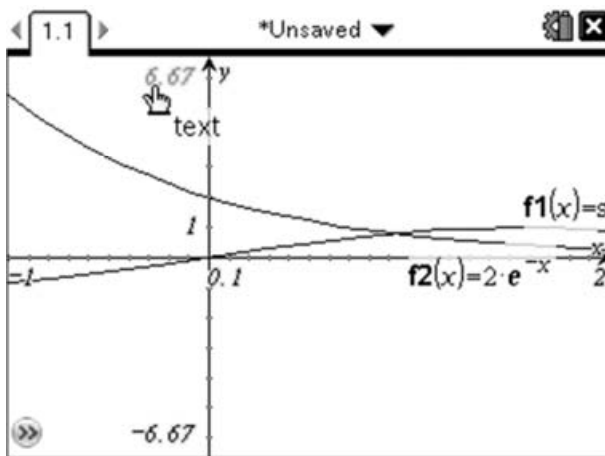

However we are wasting a lot of space with the current settings for the y-axis. We can adjust these by using the same method as above or by double clicking on the values at the axis endpoints (i.e. 6.67) and typing a different value. Double click on the YMax value (6.67) and change this to, say, 6 then press (enter). Then repeat this for the YMin value (-6.67) and change this to, say, -2 and press (enter). This should give you the following:
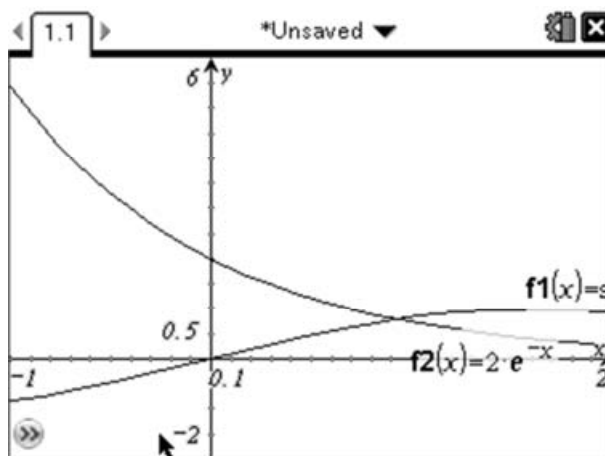

which can be used as the basis of your sketch.

The second part of the question asks us to solve the equation $f(x) = g(x)$. As the domain of these functions is restricted we are only interested in the solution that can be seen, not the one we identified earlier near x=3.

There is no analytical method for solving the equation $\sin(x) = 2e^{-x}$, however there are algorithms for finding numerical solutions. We can use the TI-Nspire, which is programmed to efficiently carry out this type of algorithm, to find a numerical solution. The simplest method for finding the intersection of these two curves is to use the 'Analyze Graph' tools. In particular we will use the Intersection option. Activate the Intersection option by pressing (menu)(6)(4), then move the lower bound bar to the left of the Intersection then click ((✸)) and move the upper bound bar to the right of the intersection and click:
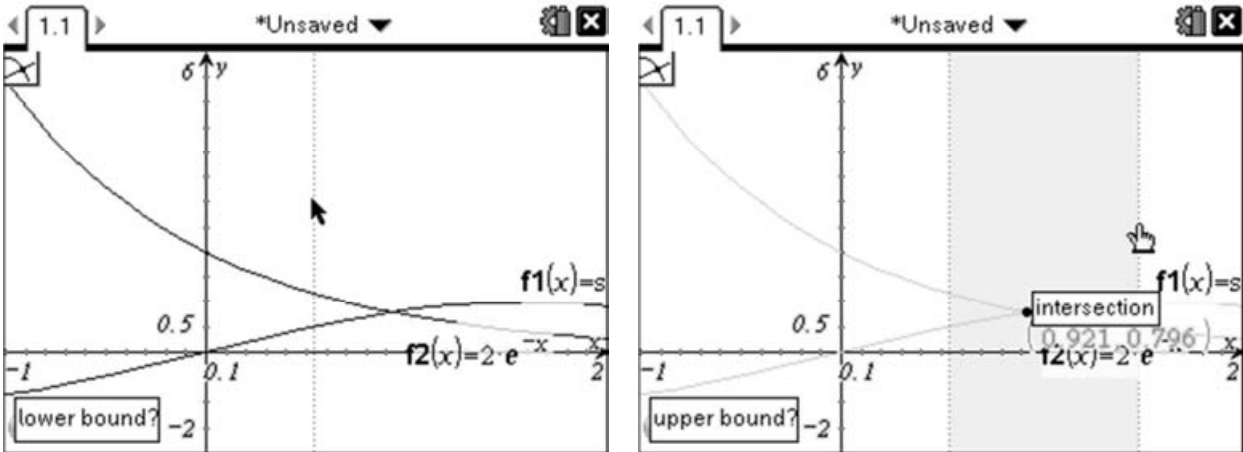
This will cause the TI-Nspire to add a point at the intersection of the two functions and show the co-ordinates:
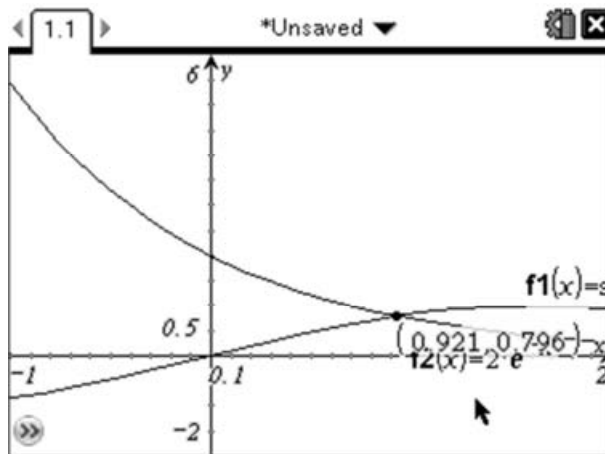

By default, the co-ordinates will be given to 3 significant figures. If we needed more we could select the relevant co-ordinate by moving the cursor over the co-ordinate then right-clicking ((ctrl)(menu)) and selecting the attributes option. This gives us a dialogue box which can be used to change the precision of a number. Adjust the value by pressing left or right, then press (enter) to store the change:
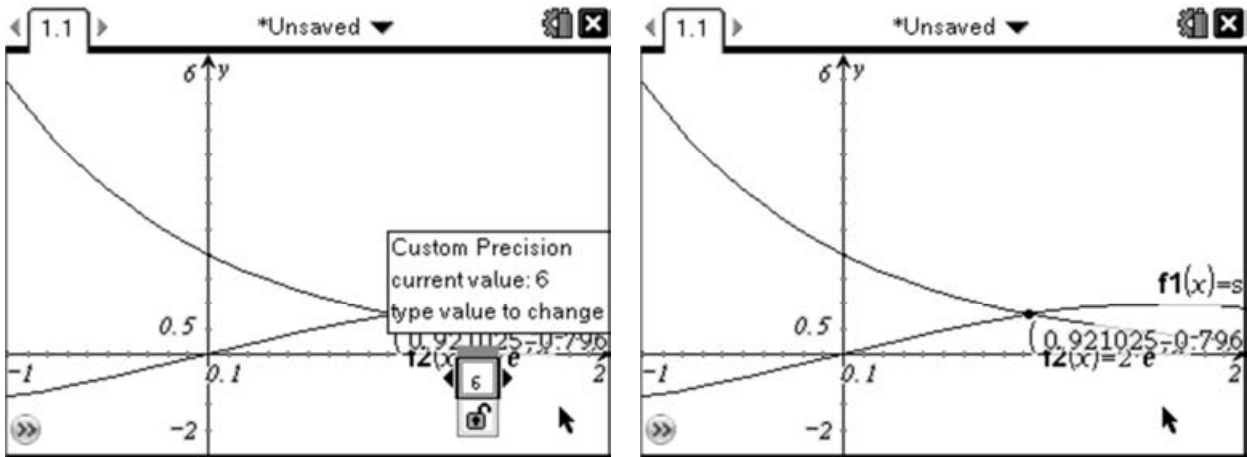
This therefore gives us the solution to the equation $f(x) = g(x)$ when $x = 0.921025$

Markscheme:

Sketch of the Graph:
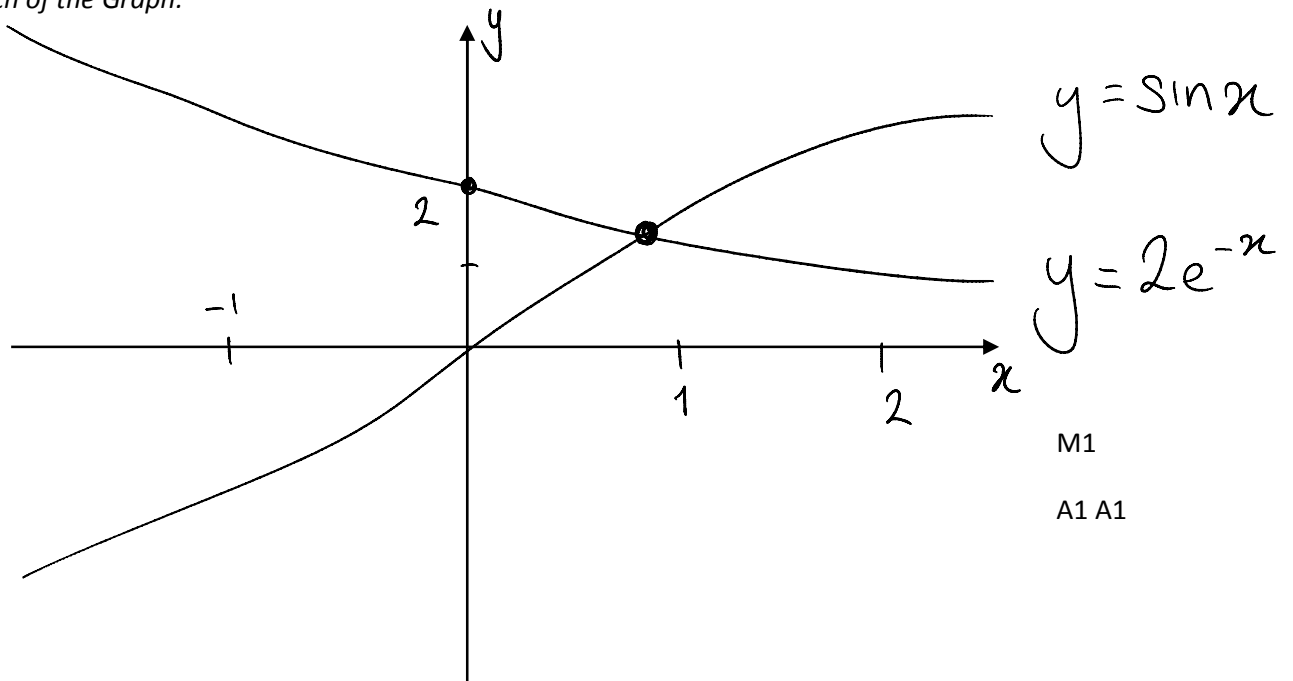
M1

A1 A1

$x = 0.921 \, (3sf)$ A1

Notice here whilst all the work was done on the GDC, you would not obtain full marks if you only wrote down the answer. It is essential that you include a reasonably accurate sketch in your answer, which should include all the relevant points on the graphs (e.g. intersections of graphs, intersections with axes and relevant maximum, minimum and inflexion points).

Arithmetic Sequences:

> **Question:**
> Let $u_n = 2n - 1, for \ n \geq 1$.
>   i) Write down the first 3 terms of the sequence
>   Find
>   ii)
>
> $$\sum_{1}^{35} 2n - 1$$
>
>   iii)
>
> $$\sum_{36}^{80} 2n - 1$$
>
> [6 Marks]

This question is about working with arithmetic sequences and their related series. The first part simply asks us to write down the first 3 terms of the sequence. This shouldn't require the use of a GDC as we are only asked to find:

$$u_1 = 2 \times 1 - 1 = 1$$

$$u_2 = 2 \times 2 - 1 = 3$$

$$u_3 = 2 \times 3 - 1 = 5$$

However we can use the TI-Nspire to explore sequences from a variety of perspectives (and sequences which are much more complicated than this example!). We will start by looking at the sequence graphically. The first thing we need to do is insert a Graphs page (Press (ctrl)(doc▾) then select 'Add Graphs'). By default, the graphing page is set up to draw Cartesian graphs, i.e. those of the form y=f(x). So to plot our sequence we need to change our graphing mode to 'plot sequences'. We can do this by pressing (menu)(3)(5)(1), which will change the function entry line to look like this:
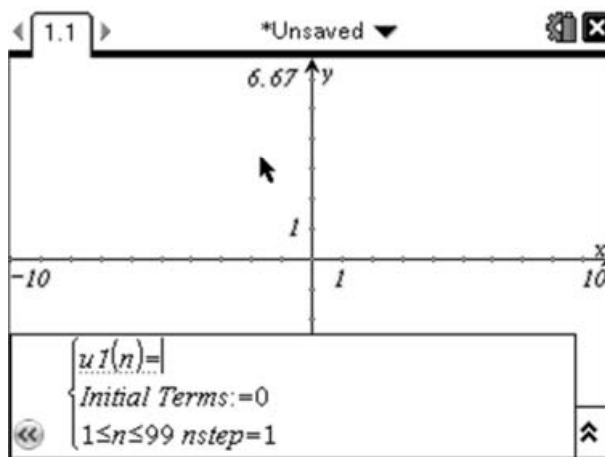

We can enter our sequence as $u1(n) = 2n - 1$. However, we must change the first term to be the correct value, as it doesn't calculate this for us. Being able to change the initial value is very important for some sequences, like the Fibonacci sequence which you could enter here by putting $u1(n) = u1(n - 1) + u1(n - 2)$. As this requires two starting values, we enter the Initial Terms as 1,1((1)(,)(1)).

Changing those initials terms to different values gives you a related set of sequences known as Lucas[1]numbers).

For our sequence, the Initial Terms should be entered as 1 because $2 \times 1 - 1 = 1$. Pressing (enter) will then plot the sequence:
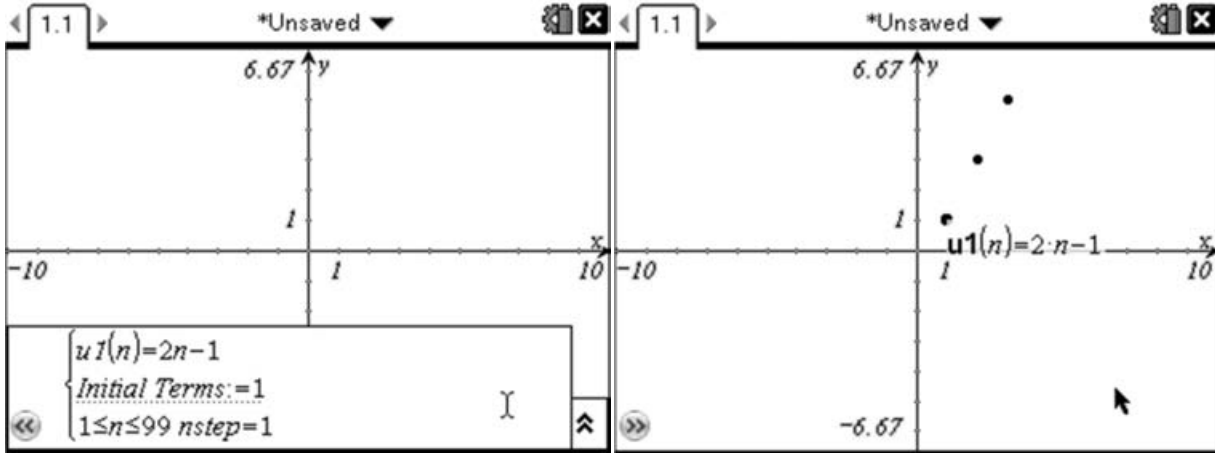

We now have a graph of the points in the sequence, with the x-axis representing the term number and the y-axis the value of that term. We can zoom out and look at the sequences in many different ways, but often it is useful to see a table of values, which we can do by pressing $\text{(menu)}\text{(2)}\text{(9)}$. This will split the screen and display a table of values on the right hand side:
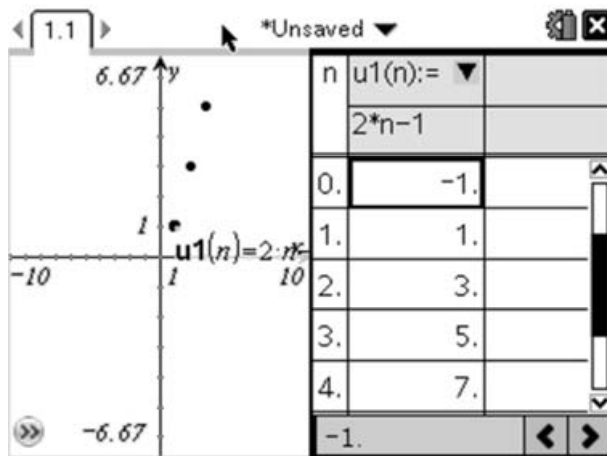
This corresponds to the values we found previously, and we can scroll through this list to look at other values in the sequence. In addition, now that we have defined the sequence, we can find any term in the sequence by going to a calculator page ($\text{(ctrl)}\text{(doc}\blacktriangledown\text{)}$ and select 'Add Calculator') and typing $u1(77)$ to find the 77th term etc.

---

[1]For Lucas Numbers the first term is 2, and the second term is 1. The third term is still the sum of the previous two terms and so on…
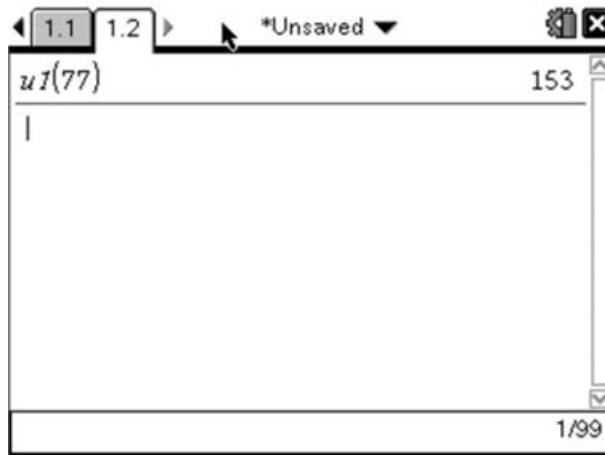
Now the slightly more difficult part of this question is related to the summation of these sequences. We know there is a formula for the sum of an arithmetic sequence given by:
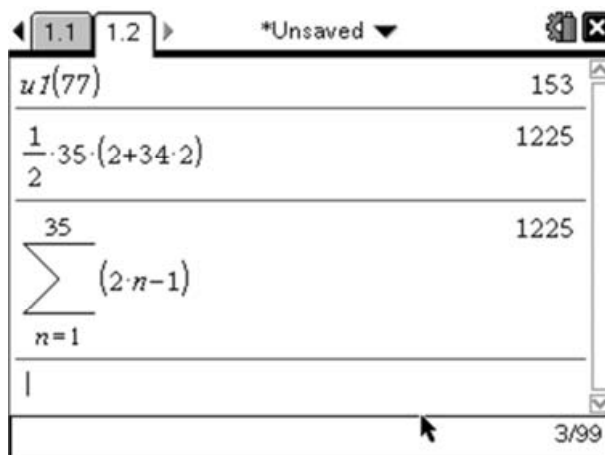
$$S_n = \sum_1^n u_n = \frac{1}{2}n(2u_1 + (n-1)d)$$

For our sequence, $u_1 = 1$ and $d = 2$, so we can answer the first part by doing:

$$\sum_1^{35} 2n - 1 = \frac{1}{2} \times 35(2 \times 1 + (35-1) \times 2) = 1225$$

The TI-Nspire has a built-in template for working with summations, which we can use by pressing ⊕ and selecting the Sigma template:
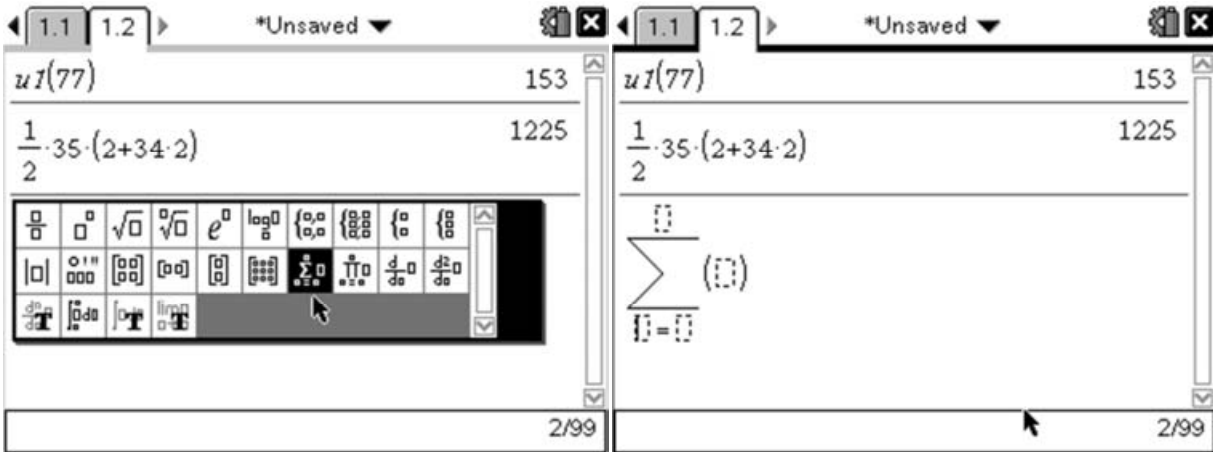
Then we can enter the relevant parts so that it looks like $\sum_1^{35} 2n - 1$, then press (enter).
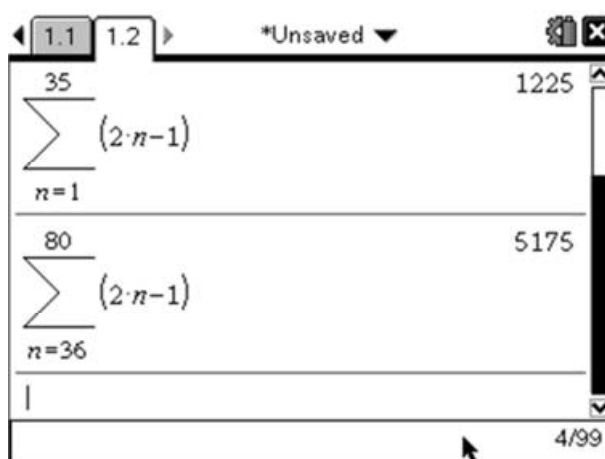

This agrees with the value we found using the summation formula.

To find the second sum we would normally find the sum to 80, then subtract the sum to 35:

$$\sum_{36}^{80} 2n - 1 = \sum_{1}^{80} 2n - 1 - \sum_{1}^{35} 2n - 1$$

$$\sum_{36}^{80} 2n - 1 = \frac{1}{2} \times 80(2 \times 1 + (80 - 1) \times 2) - \frac{1}{2} \times 35(2 \times 1 + (35 - 1) \times 2) = 6400 - 1225 = 5175$$

Using the same template as we used before we can do the sum between different limits. Press 'up' twice to select the previous summation that we entered, and press (enter) to copy it down to the entry line. Move the cursor to the lower limit where it currently says 1 and change this value to be 36, and change the 35 to 80. Then press (enter) to calculate the value.

This again agrees with the value we calculated manually. These techniques can also be used with geometric sequences and other non-linear sequences and series.

Markscheme:

$u_n = 2n - 1, n \geq 1$

$u_1 = 2 \times 1 - 1 = 1$

$u_2 = 2 \times 2 - 1 = 3$

$u_3 = 2 \times 3 - 1 = 5$ A1

Evidence of using the sum of an AP

e.g. $S_n = \frac{1}{2}n(2u_1 + (n-1)d)$ or $S_n = \frac{1}{2}n(u_1 + u_n)$ (M1)

$\sum_{n=1}^{35} 2n - 1 = 1225$ A1

Correct Calculation of $\sum_{n=1}^{80} 2n - 1$ (A1)

Evidence of Subtraction e.g.

$\sum_{n=36}^{80} 2n - 1 = \sum_{n=1}^{80} 2n - 1 - \sum_{n=1}^{35} 2n - 1 = 400 - 1225$ (M1)

$\sum_{n=36}^{80} 2n - 1 = 5175$ A1

As the method marks are implicit in these questions, full marks would be awarded for a correct answer, however it is still sensible to try and explain the process in your answer to make it clear how you have approached the question. If your answers are wrong, but you used a correct shown method, you would at least receive method marks. Without any method shown, if you answers are wrong, no marks can be awarded.

Features of Graphs:

Question:

The velocity of a fairground ride is modelled by $v(t) = \frac{t^3}{10} - \frac{5t^2}{2} + \frac{63t}{5}$ in ms^{-1},

where t is in seconds and $0 \leq t \leq 18$

Find:
i) the acceleration of the ride when $t = 9$
ii) the maximum speed of the fairground ride
iii) the time when the ride momentarily stops
iv) the total distance travelled by the ride

[6 Marks]

In this question the mathematics is embedded in a scenario about kinematics. In questions like this the emphasis is on your ability to correctly interpret the question. So on the calculator papers there is an expectation that you use your GDC to answer the question.

The key to answering kinematic questions is understanding the relationship between acceleration, velocity and displacement:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right)$$

So part (i) of this question is asking for us to find the gradient of the curve at the point $t = 9$. We could do this manually by differentiating the function and substituting in $t = 9$. But given that we have several aspects we need to look at, it makes most sense at the stage to begin by drawing the graph of the velocity function. To do this we need to replace the independent variable t with x, so we now have:

$$f(x) = \frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}$$

To graph this function on the TI-Nspire we must first create a new page ((ctrl)(doc▾)) and select a graphs page. Then enter the function in the question as **f1.** Pressing (enter) will then draw the graph.
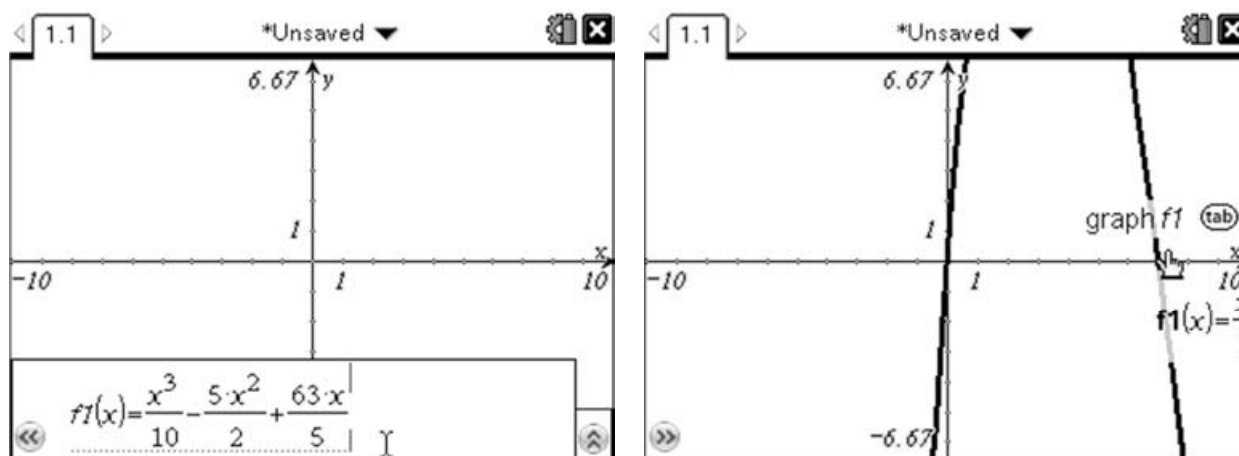

The default scaling for this graph hides the shape of the graph so we must begin by changing the window settings by pressing (menu)(4)(1) which will open the window settings. As $0 \leq t \leq 18$, we can adjust the window settings so that $-2 \leq x \leq 19$, which means we will be able to see all the relevant features.
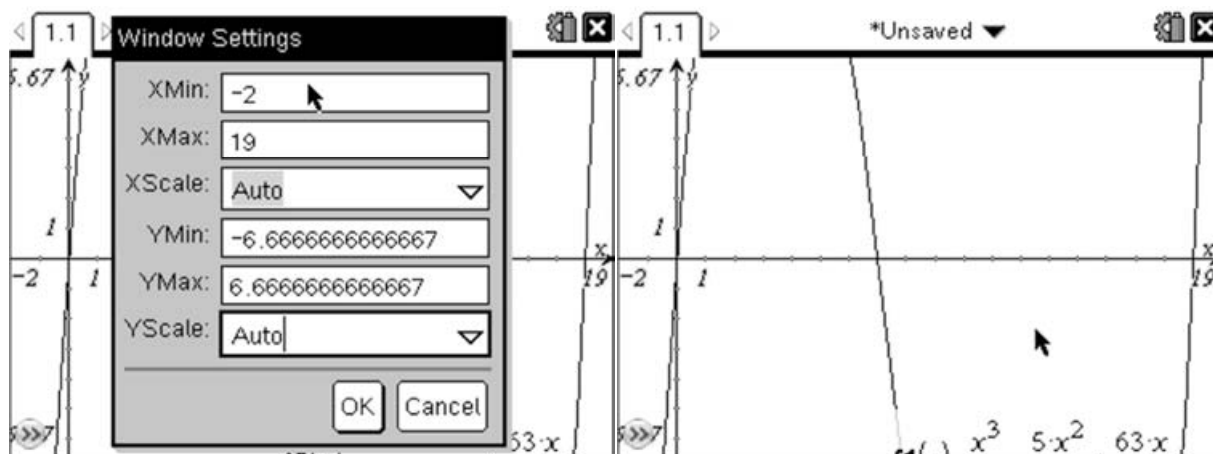
We could have tried to guess the appropriate values for the y-axis, but in this case it makes more sense to adjust this by adjusting the axis. Move your cursor so that it is over one of the tick marks on the y-axis (which should give you the ⟲ symbol). Then hold down the (⇧shift) key and pull the axis towards the centre. Holding down the (⇧shift) key, means that only the selected axis resizes.
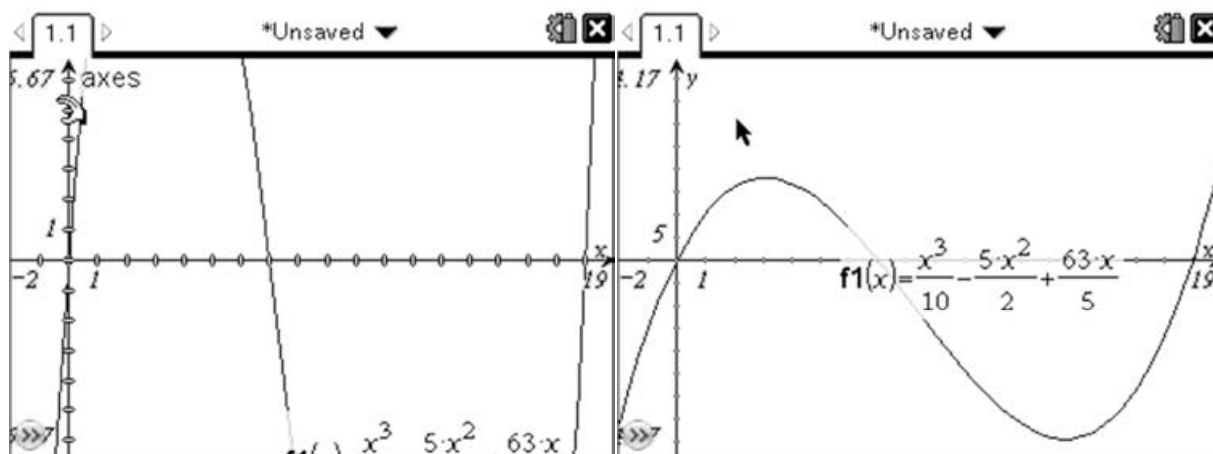
You may at this point also want to move the label for the function out of the way. This can be done by moving the cursor over the label. Then click and hold ⟐ in the centre of the touchpad, then move the label to a more convenient location and press ⟐ to drop the label.
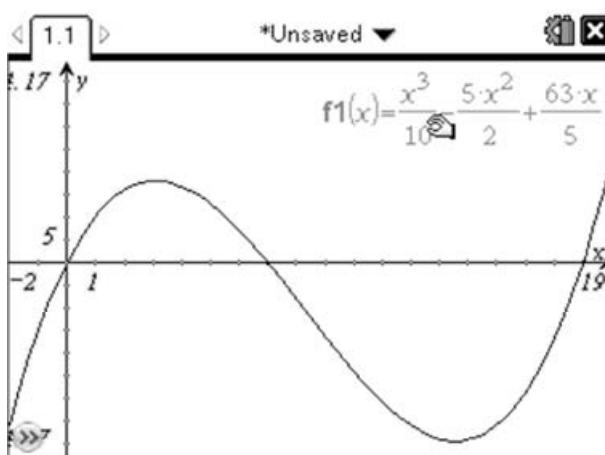
Now that we have our graph, we are ready to look at the various parts of the question.

In (i), the question asks us to find the acceleration when $t = 9$. This means we need to find the gradient of the velocity function when $x = 9$. The TI-Nspire has a built in function for finding the gradient at a point which can be found on the Analyze Graph menu (menu 6 6). You can now chose your point by just clicking somewhere on the graph. However in this case we want an exact point where $x = 9$, which we can achieve by pressing the left bracket key then typing in the x-coordinate followed by enter. This trick can be used with many of the Graphs and Geometry tools.
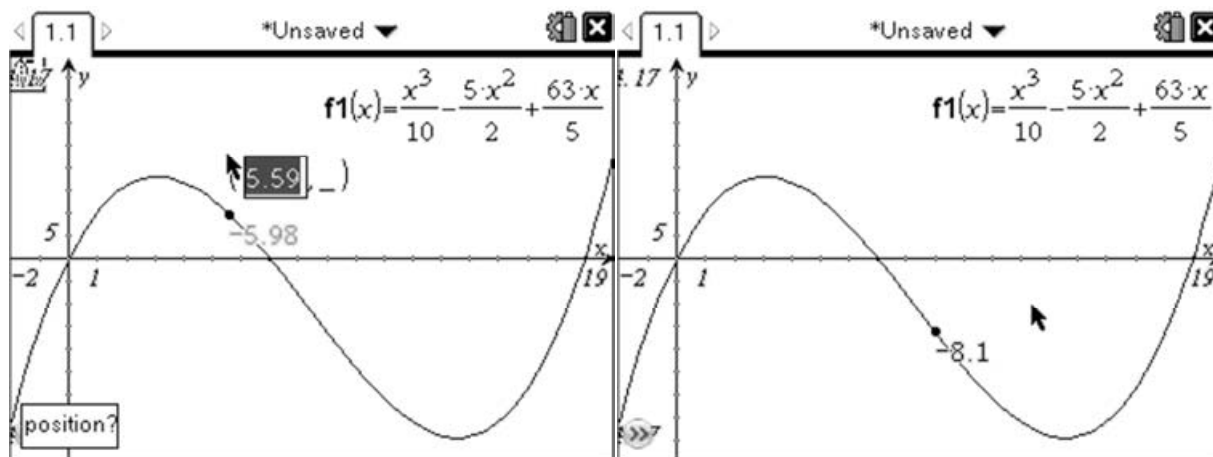
This gives us a gradient of -8.1, which equates to an acceleration of -8.1ms^{-2}, or a deceleration of 8.1ms^{-2}.

In (ii), the question asks us to find the maximum speed. As speed is a scalar quantity (i.e. the direction is not important) this means we are looking for the turning point on the graph with the largest $|y|$ value. So this could be either the maximum or the minimum depending on which is the larger. There are two ways we can tackle this:

Trace:

The TI-Nspire has a useful feature in that when you trace along a curve it will automatically 'lock' on to any interesting points on the curve. To trace the curve press (menu 5 1), and then use the touchpad to move the cursor across the curve until it locks on to the maximum and minimum points:
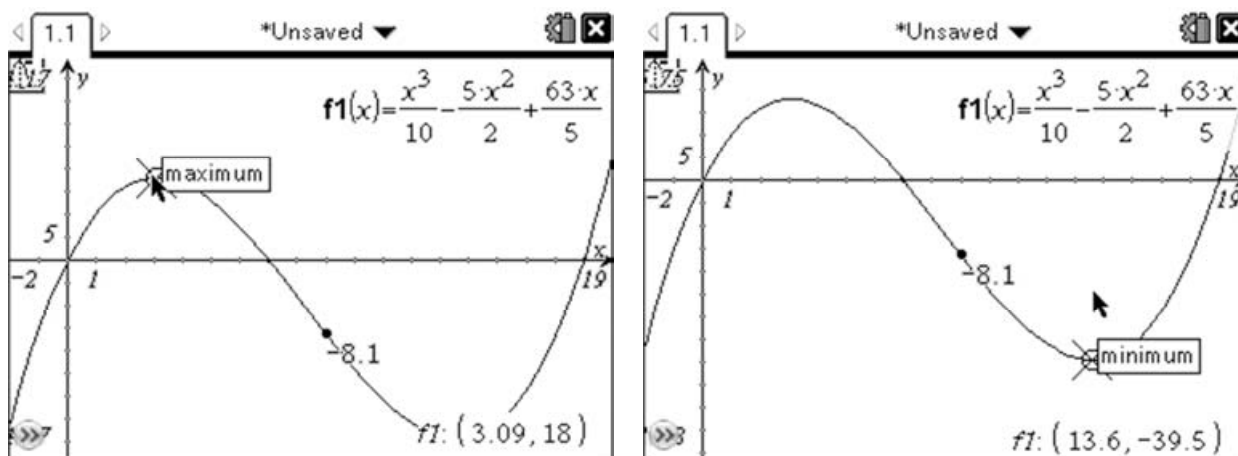
Notice that this will usually just give you the co-ordinates to 3 significant figures.

Analyze Graph:

In addition to the trace tools, there is also a set of tools to perform analysis upon the graphs. To find a minimum point we will use the Minimum option (menu)(6)(2). We will then need to select a lower bound, then an upper bound:
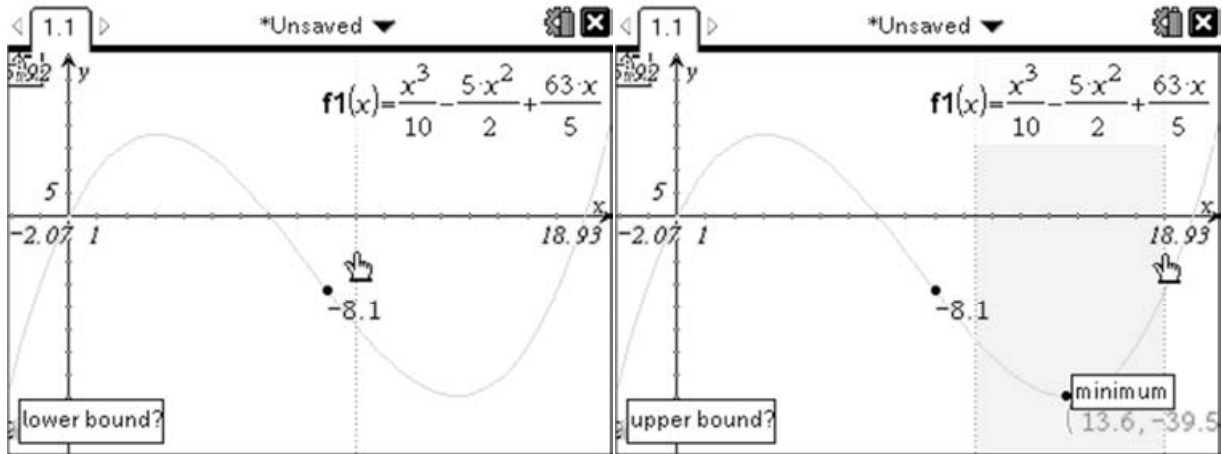

This will add a point at the minimum point with the co-ordinates labelled. If necessary you can drag the co-ordinates by moving the cursor over the label and then grabbing (either (🔾) and hold or (ctrl)(🔾)). Then you can drag the co-ordinates label to a more convenient part of the screen.

You can then repeat this by pressing (menu)(6)(3) to find the maximum value:
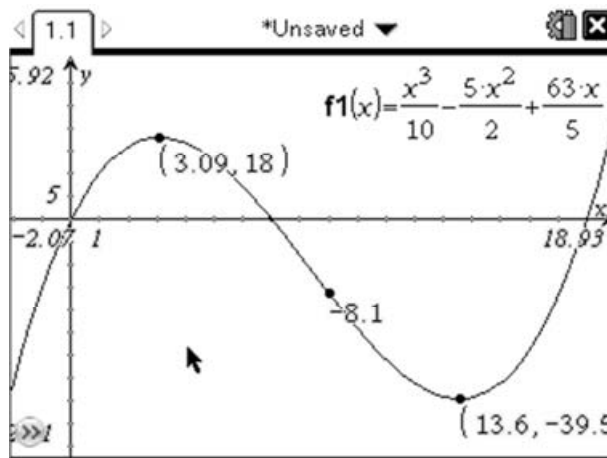
If you want a more exact answer than the 3 significant figures displayed, you can right-click ((ctrl)(menu)) on either the x or y co-ordinate then select Attributes:
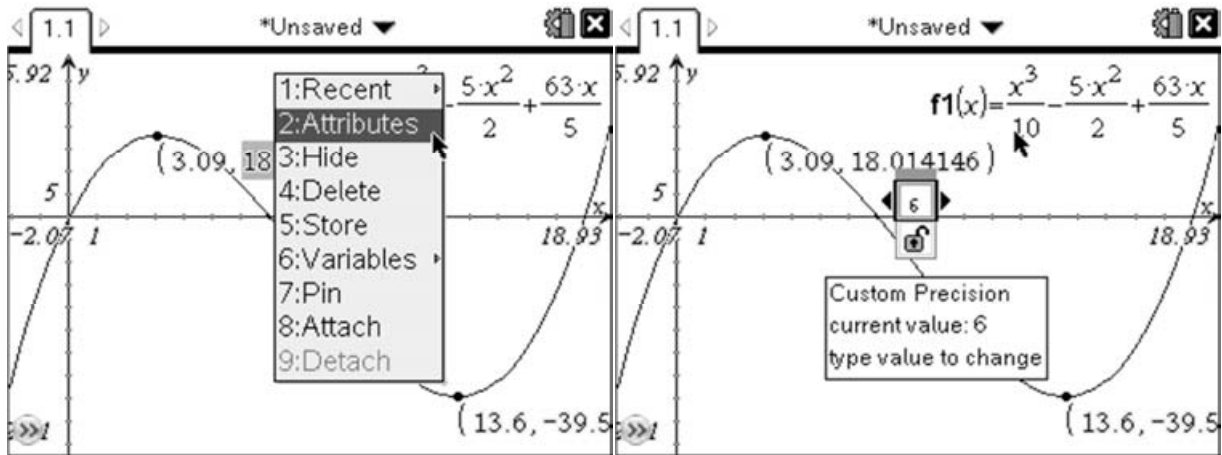
Press right to increase the number of decimal places and press (enter) to store the change. You can then repeat this for the y co-ordinate. An alternative method is to hover the cursor over the x or y co-ordinate then press the ⊕ or ⊖ to increase or reduce the number of decimal places.
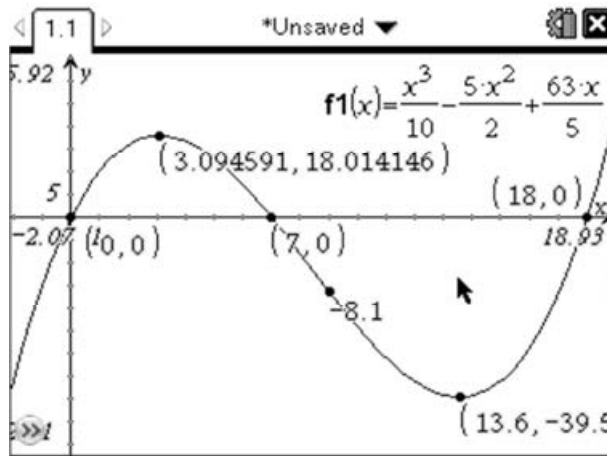
Therefore the maximum speed is 39.5ms[-1], and occurs after 13.6 seconds.

Roots:

In (iii), the question asks us when the ride comes to a momentary stop; that is, when is the velocity zero? We can use similar techniques to those above to find the roots of this function. Again the trace option should 'lock' on to the roots of the function:
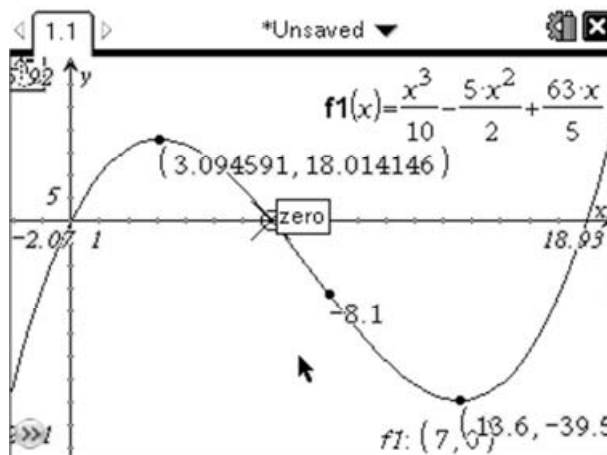
If you want to force the trace to move to a given point, you can do so by typing the required x-value. For example, for this root at x=7 press ⑦ (enter).

Alternatively we can use the Analyze Graph option, this time using the 'zero' option. Again select a lower and upper bound for each root and the TI-Nspire will add a point at the root.
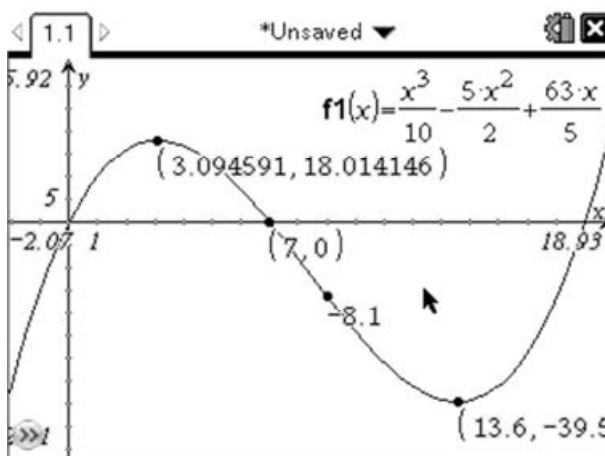
Intersection Points:

Another option that can be useful when finding roots of a function is to find the intersection of the graph of the function with the x-axis. This can be done by pressing (menu)⑦③, then selecting the graph, followed by the x-axis. This will add points for all the roots of the function. To display the co-ordinates of these points simply right-click ((ctrl)(menu)) on the points and select '7: Co-ordinates and Equations'.
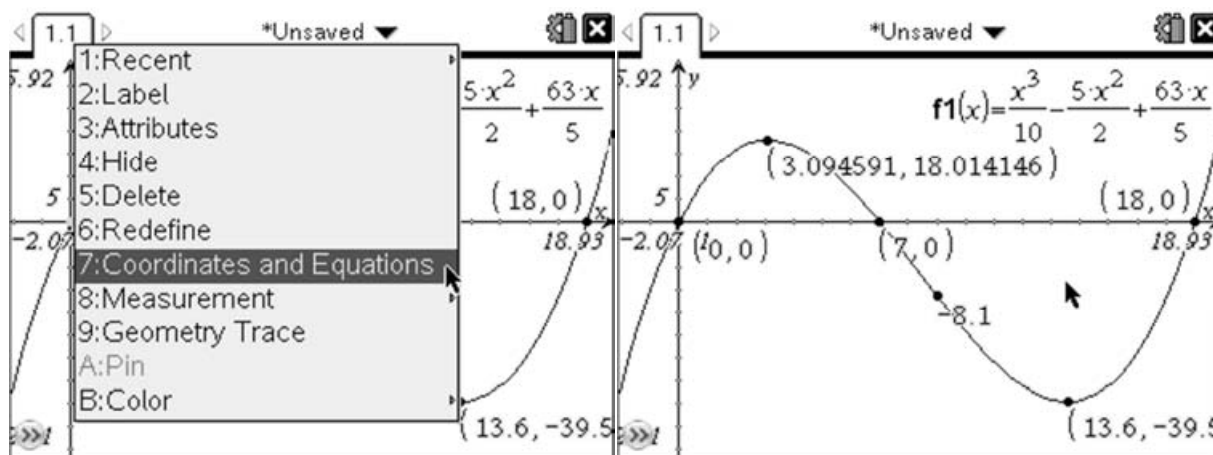
Using either of these methods, we can see that the ride comes to a momentary stop after 7 seconds.

Note: Because the Graphs section is a numerical environment it may sometimes give roots which are not exact like (0.481,4.88E-14) where 4.88E-14 is a numerical approximation to zero. This is due to the inherent inaccuracy of working with decimal numbers in an approximate numerical environment.

Part (iv) of the question asks us to find the distance travelled by the fairground ride. Since the displacement function is obtained by integrating the velocity function, this is equivalent to finding the area under the graph of the velocity function. As the question asks us for the distance travelled and not the

displacement we must calculate the negative and positive regions separately and then add them together, or find the area under the curve of the absolute value of the velocity function.

By finding the root in the previous part we now know that the area we are interested in is the integral of the curve between x=0 and x=7 and then the integral between x=7 and x=18. i.e.

$$\left| \int_0^7 \frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}\ dx \right| + \left| \int_7^{18} \frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}\ dx \right|$$

We can find these areas by again using the Analyze Graph menu, but this time selecting the Integral tool ((menu)(6)(6)). If there is only one graph on the page then you will just be asked for the lower and upper bounds for the integral. In the event that you have multiple graphs displayed you must first select the graph you are interested in.

As we have already marked the roots on the graph we can simply select them as the lower and upper bounds:
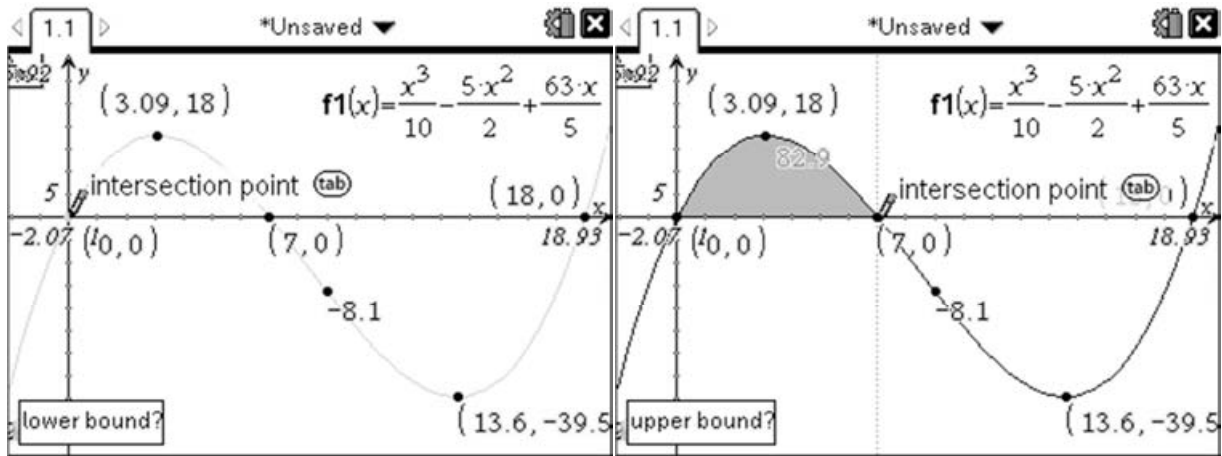
Then repeat this for the second region, which gives:
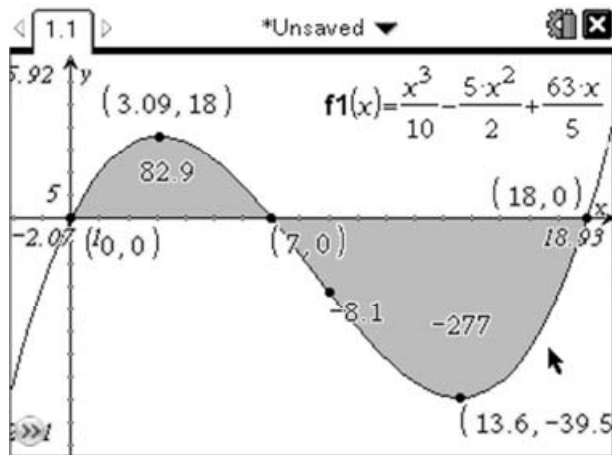
This gives us a total area of $82.9 + 277 = 359.9$, therefore the ride has travelled a total of 360m (3sf).

Calculator Page:

An alternative approach to drawing using the graphs is to make use of the various functions available on the Calculator page.

The best way to do this is to start by defining the function as f(x). We can do this by pressing $\boxed{F}①\boxed{X}①$ $\boxed{ctrl}\boxed{e^x}$ which will enter f(x):= after which we can enter the function:
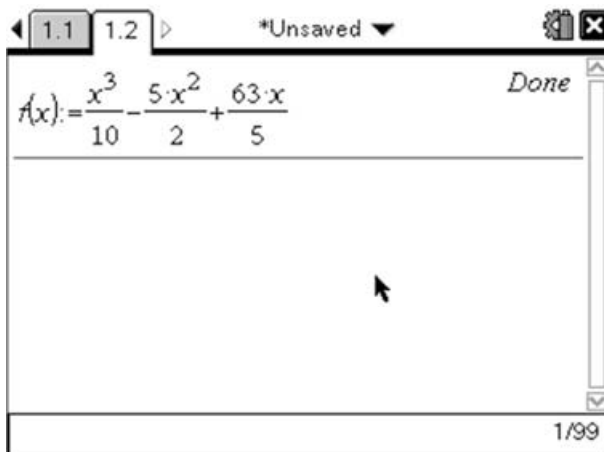
We can now find the acceleration by finding the derivative at a point using the derivative template ($\boxed{menu}④①$). In the wizard, set the value to be 9 and press $\boxed{enter}$. Then either enter the function or just put f(x) in the gap in the template:
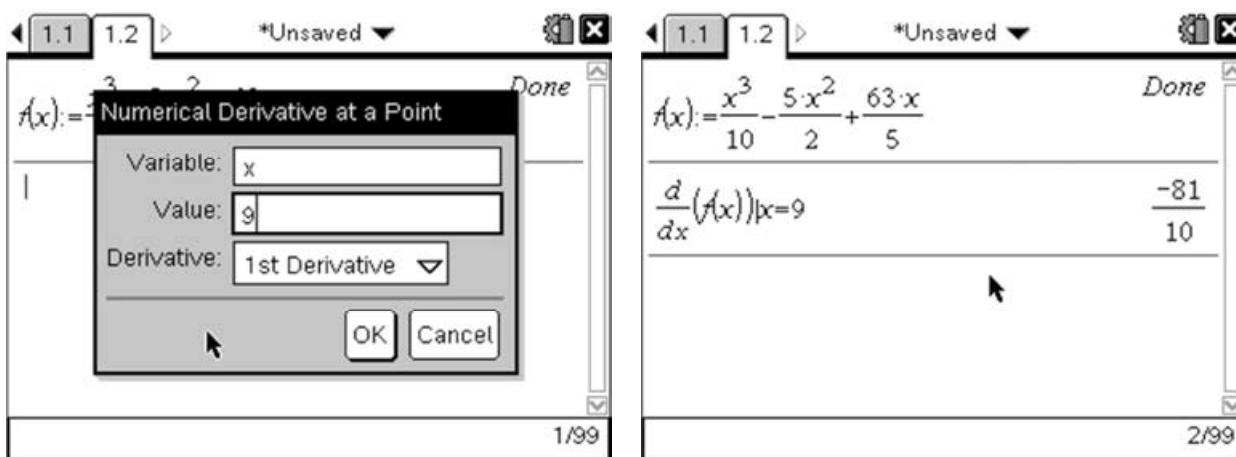
This gives us an acceleration of -8.1ms^{-2} (i.e. deceleration) when t=9.

To find the maximum point of the function we can use the nfMax function ($\boxed{menu}④⑥$).

$$nfMax\left(\frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}, x\right)$$

This will return the x co-ordinate where the function is maximal. The problem with this is that it will attempt to return the maximum value across the whole domain, which for this function is infinite!
To deal with this we would need to restrict the interval of interest, which in general is best done by graphing the function.

To consider the maximum value with the interval restricted between **a** and **b**, we would enter:

$$nfMax\left(\frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}, x, a, b\right)$$

If you only want to restrict the lower bound then simply omit **b.**

We can now find the maximum point as follows:
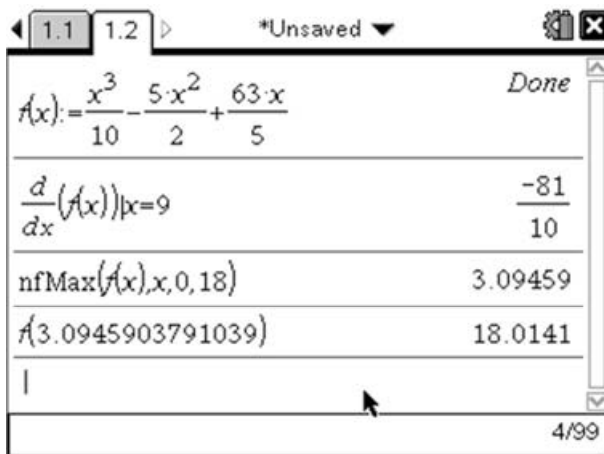
To do this press (menu)(4)(6) to enter the nfMax function, then type f(x),x,0,18 using the letter keys at the bottom of the handheld. To find the y co-ordinate simply type [F](①) then press 'up' to select the x co-ordinate 3.09459. When this is selected press (enter) to copy down the value and then (enter) again to evaluate the function.

We can use a similar approach to find the minimum value as shown below:
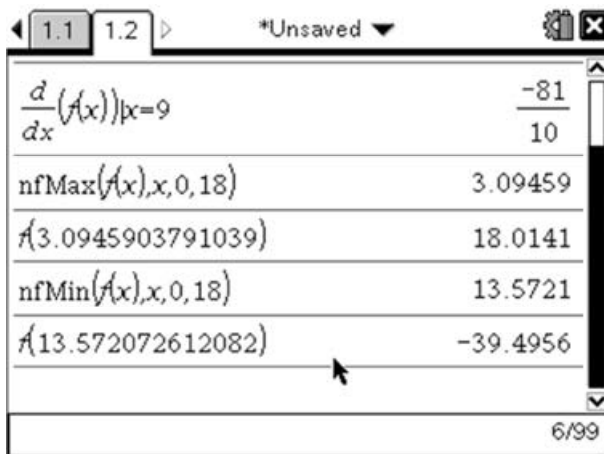
There is no equivalent function to find the roots of a function. However, we could use the nSolve function to find the roots. nSolve has the following syntax – nSolve(equation, variable to be solved for, lowerbound, upperbound) although you can leave off either or both the upper and lower bounds.

To find the roots of this function we will start by asking it to find any root using the following command:

$$nSolve(f(x) = 0, x)$$ - (menu)(3)(1) then enter **f(x)=0,x)** and press (enter)
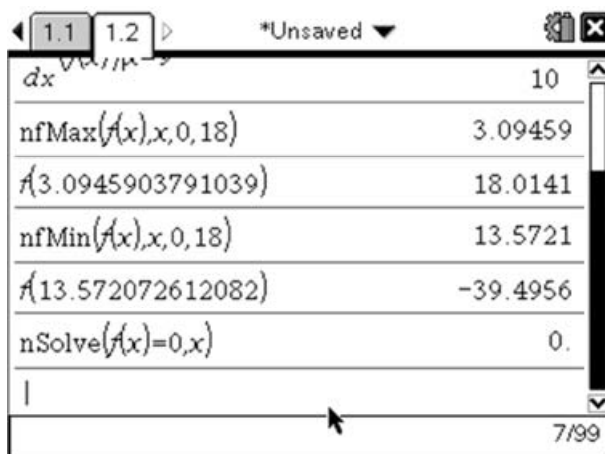
To find another solution we need to limit the interval in the same way as we did for nfMax function. Given that the maximum point is to the right of this first root it seems likely that if there is another it is to the right of this root. So we will use the first root as the lowerbound to find the second root.

$nSolve(f(x) = 0, x, 0)$ - (menu)(3)(1) then enter **f(x)=0,x,** then press 'up' to highlight the previous solution and press (enter) to copy the value down, then press (enter) to run the command.
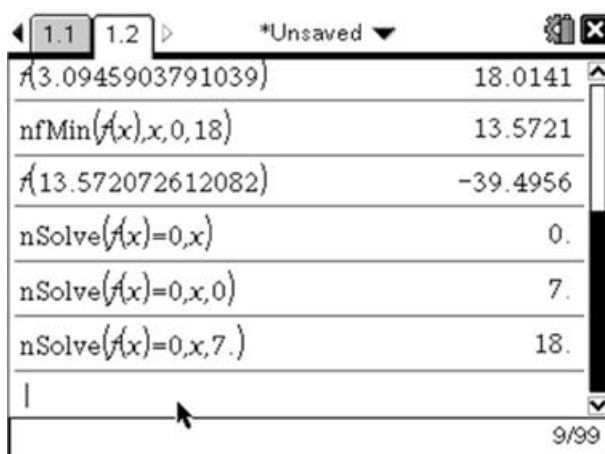
The advantage of this technique is that it works for most well-defined functions. However, as this function is a polynomial, we could also use the polynomial solver to find its roots. The polyRoots function allows us to find the root of any polynomial. This tool can be accessed by pressing (menu)(3)(3)(1), which will start the wizard. Change the degree of the polynomial to be 3, and enter the co-efficients of the cubic. This will return a function which looks like:

$$\text{polyRoots}\left(\frac{x^3}{10} - \frac{5x^2}{2} + \frac{63x}{5}, x\right)$$

If you prefer, you can enter this function manually by simply typing 'polyroots' and then the function using the letter keys at the bottom of the keypad.
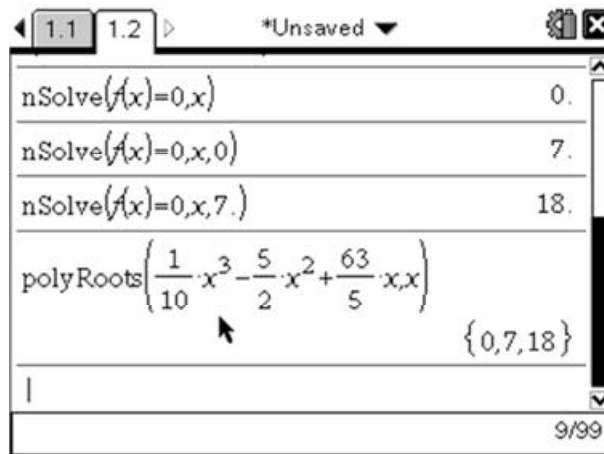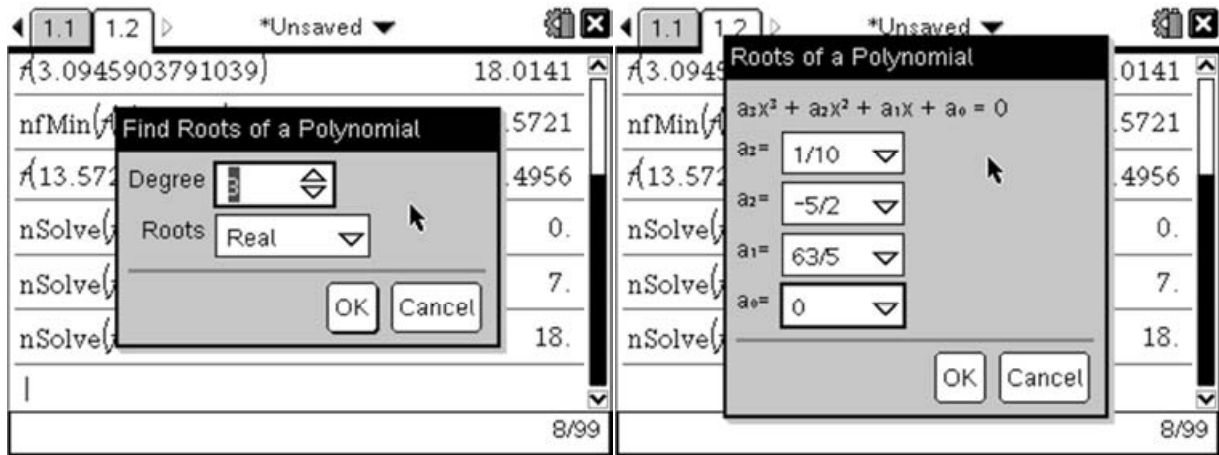
These approaches can be useful if you are looking for greater numerical accuracy, or want to carry out further calculations based upon these values. Often it can be useful to try a combination of approaches, starting graphically to get a feel for the shape of the function, before trying a numerical approach.

We can also find the area by using the Numerical Integral function, which has the following format:

$$\int_{\square}^{\square} \square \, d\square$$

From the graph we know that the area is bound between the second and third roots. We can access the Numerical Integral function by pressing (menu)(4)(2) or by pressing (mħ) which gives us access to the template menu. We can calculate the area by entering:
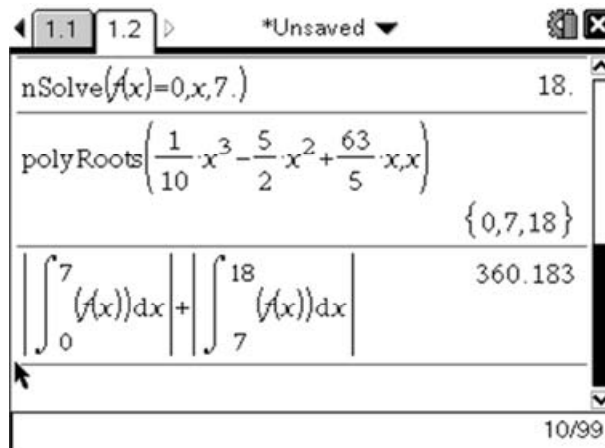
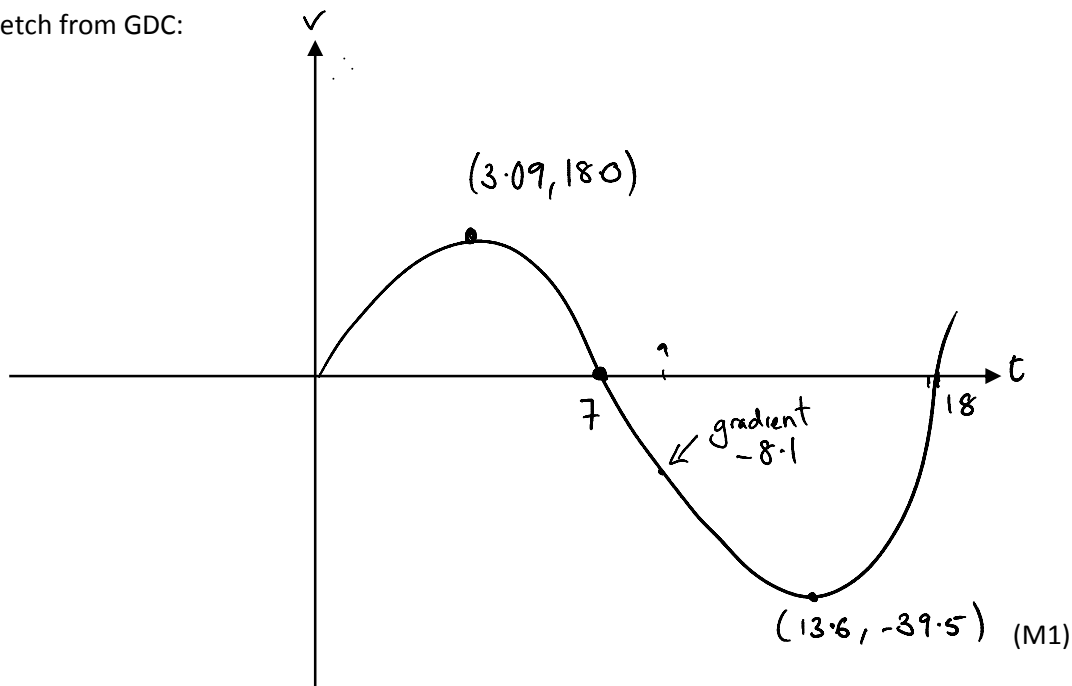giving us an area of 360.183, which corresponds to the distance travelled.

Alternatively we could have found the integral of the modulus of the velocity function:

$$\int_0^{18} |f(x)|\, dx$$

Markscheme:

Sketch from GDC:

(M1)
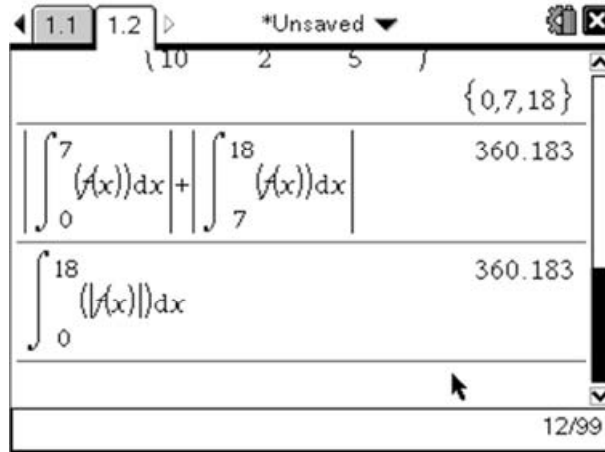
Acceleration at $t = 9$ is $a = -8.1 \text{ms}^{-2}$ A1

Maximum Speed = 39.5 ms^{-1} (3sf)   A1

Momentary Stop occurs at $t = 7$ A1

Distance travelled = $\left|\int_0^7 y\, dx\right| + \left|\int_7^{18} y\, dx\right| = 360$m (3sf)   M1 A1

Composite and Inverse Functions:

Question:

Given that

$$f(x) = x^2 + 3 \quad \text{and} \quad g(x) = \frac{x+2}{x-3} \quad (where \; x \neq 3)$$

Find:

      i) $f \circ g(4)$

      ii) $g^{-1}(6)$

                                                                                          [6 Marks]

When dealing with composite functions like the ones given in this question we always have a choice of two options. We can find $f \circ g(x)$ and then substitute in 4. Alternatively, as $f \circ g(4) = f[g(4)]$ we can find the value of $g(4)$ and then use this value in the function f.

The first approach would give us:

$$f \circ g(x) = f[g(x)] = f\left[\frac{x+2}{x-3}\right]$$

$$f\left[\frac{x+2}{x-3}\right] = \left(\frac{x+2}{x-3}\right)^2 + 3$$

Therefore

$$f \circ g(4) = \left(\frac{4+2}{4-3}\right)^2 + 3 = 6^2 + 3 = 39$$

With the TI-Nspire, we can define the functions and then work with them. To define the functions, we must first insert a Calculator page (ⒸⓉⓡⓛ ⓓⓞⓒ▾ and choose 'Add Calculator'). We can define the function $f(x) = x^2 + 3$ by pressing Ⓕ⓪Ⓧ⓪ ⓒⓣⓡⓛ ⓘⓝⓕⓘ which gives us 'f(x):=' after which we can type our definition of f(x) followed by ⓔⓝⓣⓔⓡ. We can then repeat this process for g(x) (you can use ⓒⓣⓡⓛ÷ to use the fraction template) which give us:
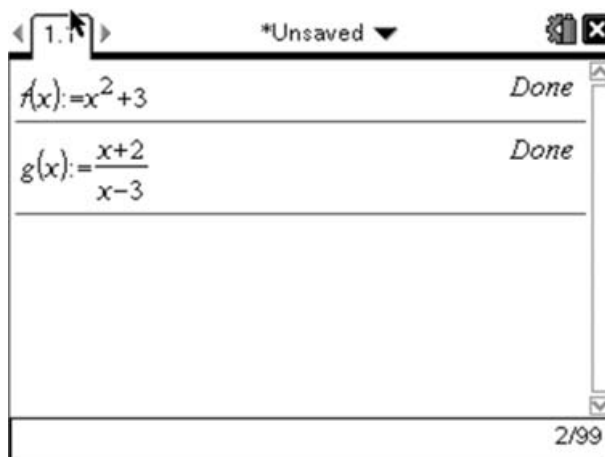
We can now find $f \circ g(4)$ by typing Ⓕ⓪Ⓖ⓪④⓪⓪ followed by ⓔⓝⓣⓔⓡ.
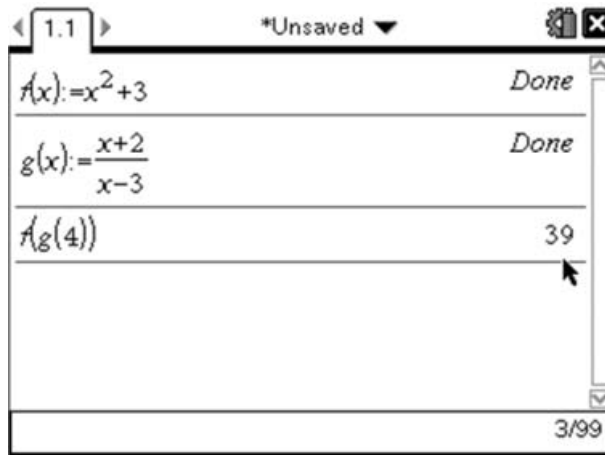
This gives us $f \circ g(4) = 39$ which agrees with our previous calculation.

Finding the inverse of $g(x)$ is a more challenging question. The traditional approach to this usually follows a pattern something like this:

$$\text{Let } g(x) = y = \frac{x+2}{x-3}$$
$$x = \frac{y+2}{y-3} \qquad \text{(Swap the x's and y's)}$$
$$x(y-3) = y+2 \qquad \text{(Then rearrange to give y=...)}$$
$$xy - 3x = y + 2$$
$$xy - y = 3x + 2$$
$$y(x-1) = 3x + 2$$
$$y = \frac{3x+2}{x-1}$$

So we have:

$$g^{-1}(x) = \frac{3x+2}{x-1}$$

And therefore:

$$g^{-1}(6) = \frac{3 \times 6 + 2}{6-1} = \frac{20}{5} = 4$$

An alternative method we could use is to realise that if we want to find $g^{-1}(6)$, then we can do this by saying $g^{-1}(6) = x$ and then applying g to both sides to give us:
$$g \circ g^{-1}(6) = g(x)$$

But $g \circ g^{-1}(6) = 6$, so we are really looking for:
$$g(x) = 6$$

We can solve this equation using the nSolve function. To do this we will use the following syntax:
$$nSolve(g(x) = 6, x)$$

This will find the first value of x for which $g(x) = 6$. Because $g(x)$ is an invertible function, this value must be unique, so we do not need to worry about restricting the range. (This could be done by entering $nSolve(g(x) = 6, x, 3,5)$ to look for a solution between 3 and 5.)

We can enter this by pressing (menu)(3)(1) to enter the nSolve function, then pressing
(G)(()(X)()) (=)(6)(,)(X)()) then pressing (enter) to execute this function. This gives us:
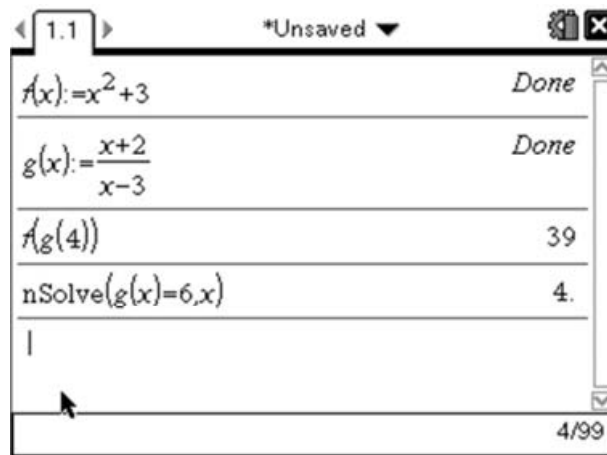
This corresponds to the value we found using the more formal method.

Markscheme:

$f \circ g(4)$

$g(4) = \frac{4+2}{4-3} = \frac{6}{1} = 6$ A1

$f(6) = 6^2 + 3 = 39$ A1

$g^{-1}(6) = x$

$g(g^{-1}(6)) = g(x)$ M1

$6 = g(x)$ A1

$\frac{x+2}{x-3} = 6$ (M1)

$x + 2 = 6x - 18$

$20 = 5x$

$x = 4$ A1

Notice that it is essential that you make your method explicit in answering questions like this. In the first part to ensure full marks you need to show the intermediate working. For the second part you need to show the mathematical steps which get you to the point where you can use the calculator to solve it (or continue to solve it by hand).

Normal Distribution:

Question:

The heights of students in class 10B are normally distributed with a mean of 165cm and a standard deviation of 20cm.

Find:
      i)   the probability that the height of a student chosen at random is more than 180cm
      ii) the value of h, given that $P(X < h) = 0.823$

[5 Marks]

In this question we are told $X \sim N(165, 20^2)$, so the first part of the question equates to finding $P(X > 180)$. We can use the graphing facilities of the TI-Nspire to explore this graphically.

Insert a Graphs page by pressing (ctrl)(doc▾), then select 'Add Graphs'. To graph the normal distribution we need to insert the normPdf function (alternatively we could use the formal definition of the normal distribution probability density function[1]), which can be accessed on the Graphs page by pressing the catalogue button (⌨) and then selecting the normPdf function (pressing N will skip you down to the functions beginning with n):
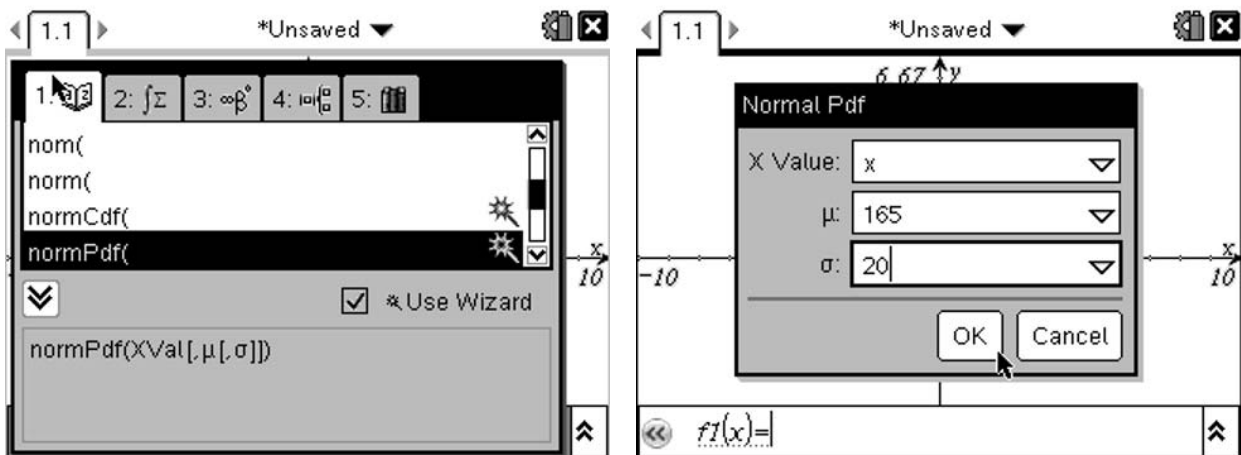
This will display the Normal Pdf wizard (if it doesn't, you need to put the tick mark in the 'Use Wizard' option). As we want to graph the function, the X value is simply 'x', and then we can use the mean and standard deviation given in the question.

---

[1] $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

This gives a very flat graph, and you may think it hasn't worked:
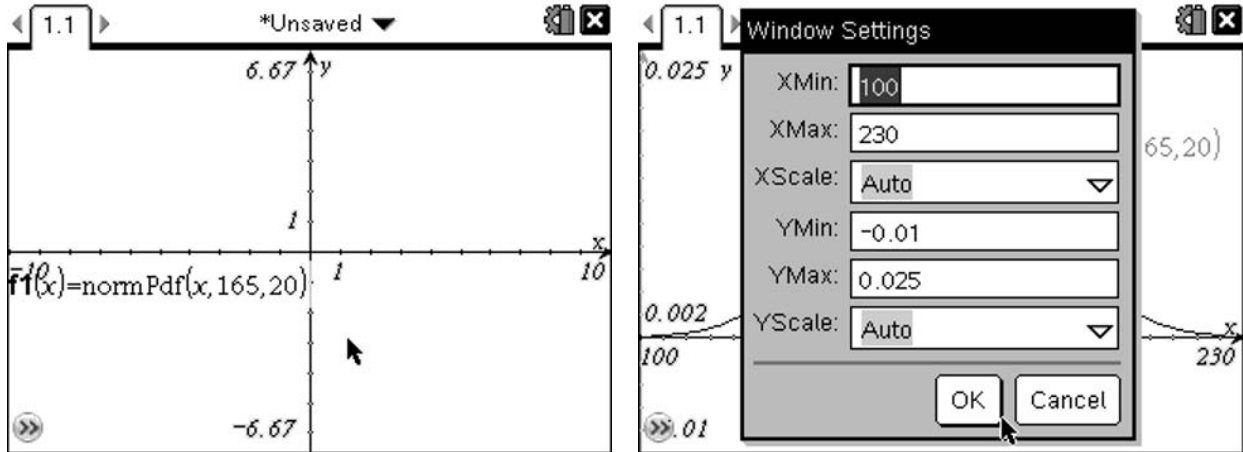
However this is just a result of not having the window scaled correctly. Adjust the window settings by pressing (menu)(4)(1), and adjust the values to those shown above. As we know that the probability of an event happening is based on the area under the graph, we can find $P(X > 180)$ by calculating the integral between 180 and infinity.

To find the numerical integral you can use the Analyze Graph menu, and choose Integral ((menu)(6)(6)). You can then select your lower and upper bounds. However as you need an exact lower bound of 180, it is better to do this by entering the exact value by pressing (()(1)(8)(0)(enter), and then moving the upper bound as far right as possible.
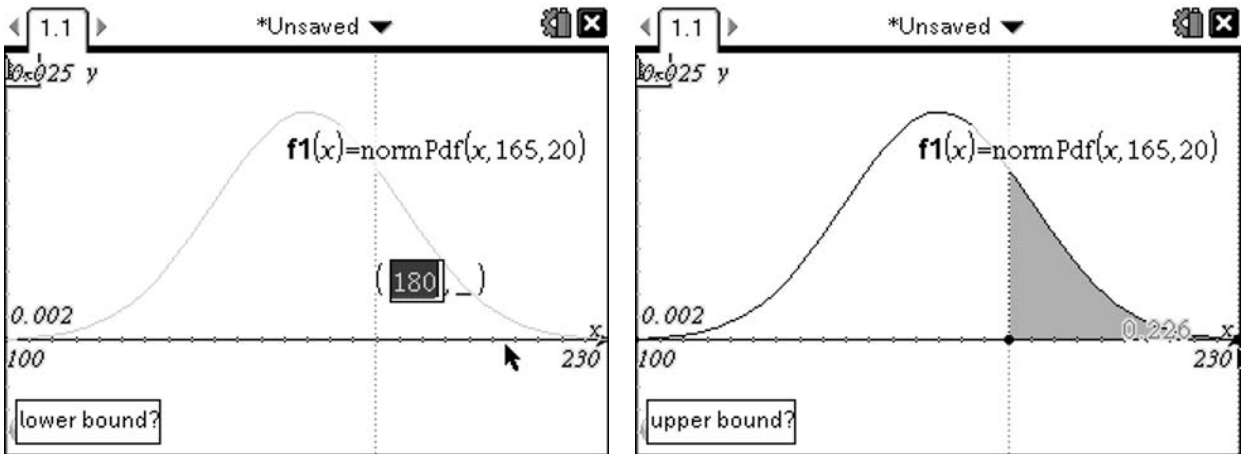
This gives us an area of 0.226, and hence a probability of approximately 0.226.
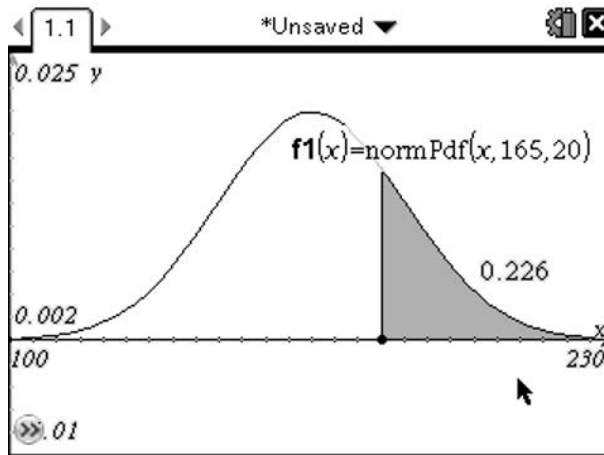
Normalising:

The traditional method to answer this question would be to normalise. This gives us:

$$Z = \frac{X - \mu}{\sigma} = \frac{180 - 165}{20} = 0.75$$

We can then use the normCdf function which allows us to calculate the probability of for a normal distribution between an lower and upper bound. For this question we want the probability of $P(Z > 0.75)$. As there is no upper bound on this problem, we will use a very large number. The TI-Nspire defaults to the largest number it can cope with, which is 9×10^{999} as a numerical approximation to ∞.

Aside: Given that the normal distribution has an asymptote at $y = 0$, the probability of an event a long way from the mean approaches zero very quickly as we move off towards infinity, i.e. $P(X > x) \to 0$ as $x \to \infty$. We can hence use a numerical approximate for infinity to obtain an accurate value for the probability to a large number of significant figures as long as the standard deviation remains relatively small (i.e. less than 20 digits or so!). If you want to test this try calculating normCdf with different upper bounds and compare the values… In fact, for this particular example, we are accurate to 14 decimal places with an upper bound as small as 8.

So we want we want to enter the function $normCdf(0.75, 9 \times 10^{999}, 0, 1)$. We can do this by inserting a Calculator page by pressing (ctrl)(doc▾) then selecting 'Add Calculator'. Now we can enter the normCdf function by pressing (menu)(5)(5)(2) which will display the distribution wizard:
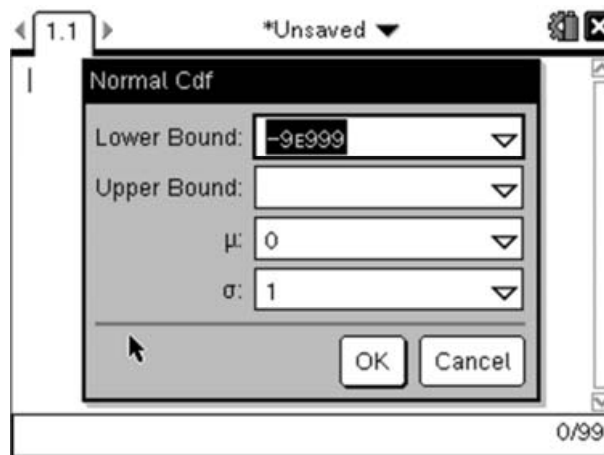
We can now enter the lower bound as 0.75 and then press (tab) to move the next option then enter the upper bound as $9 \times 10^{\wedge}999$ by pressing (9)(EE)(9)(9)(9) (or any reasonable upper bound). Then press (enter) to close the dialogue and execute the function:
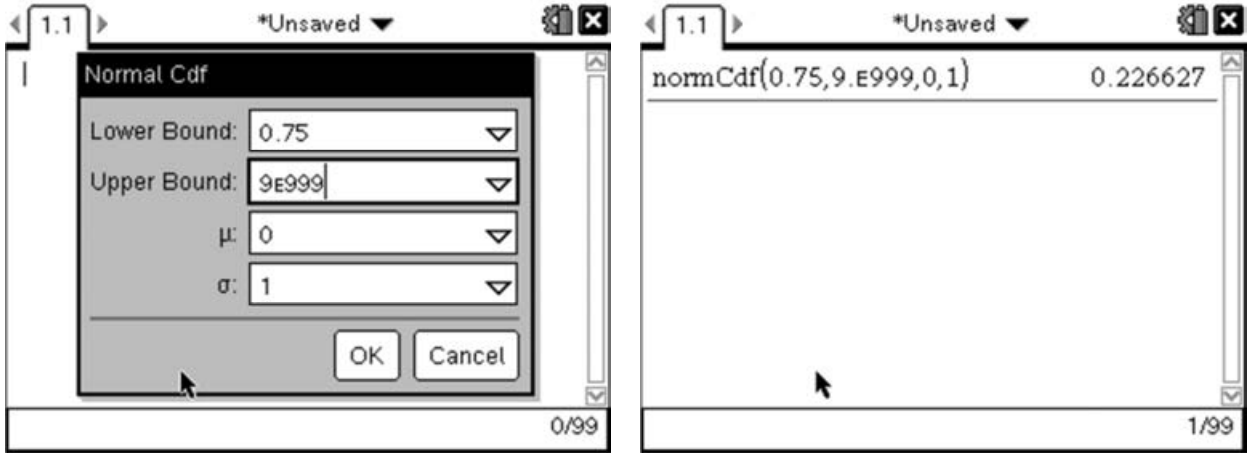

However you may have noticed that the dialogue box had options for entering the mean and standard deviation, meaning that we don't need to first normalise. Let's repeat this calculation but this time using the lower bound as 180, and including the mean as 165 and standard deviation as 20.
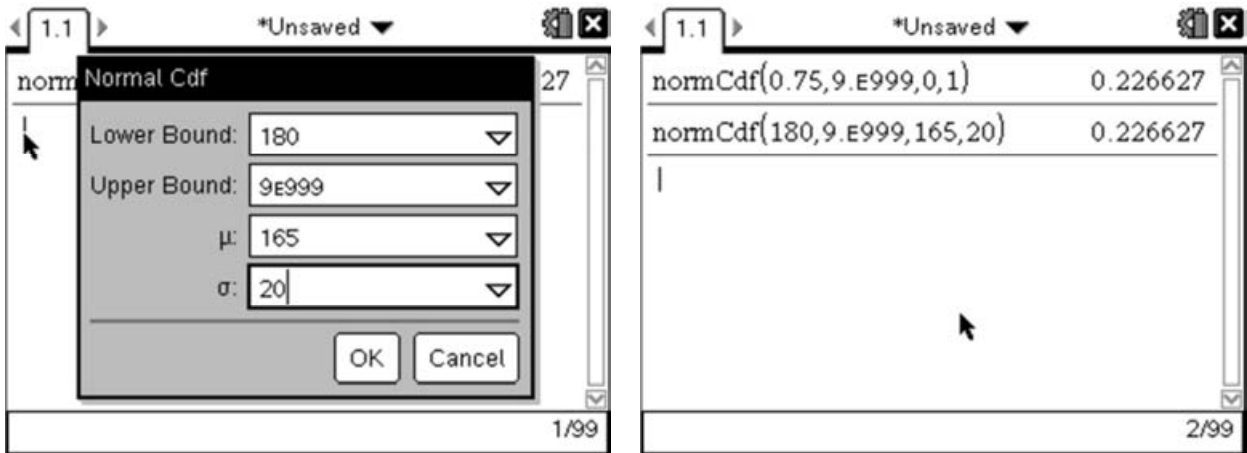

Notice that this gives us the same result and rounded to 4 decimal places it also agrees with the value we found using the tables.

The TI-Nspire has an inverse normal (invNorm) function that we can use in this situation. We can start the invNorm wizard by pressing (menu)(5)(5)(3):
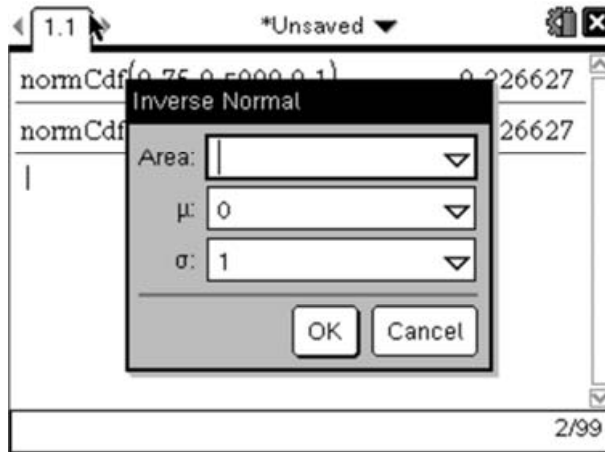
As we know, the probability in the normal distribution is given by the area under the curve, so in our case the area is the same as the probability and should be entered as 0.823. We can then enter the mean and standard deviation to save the need to undo the normalisation process.
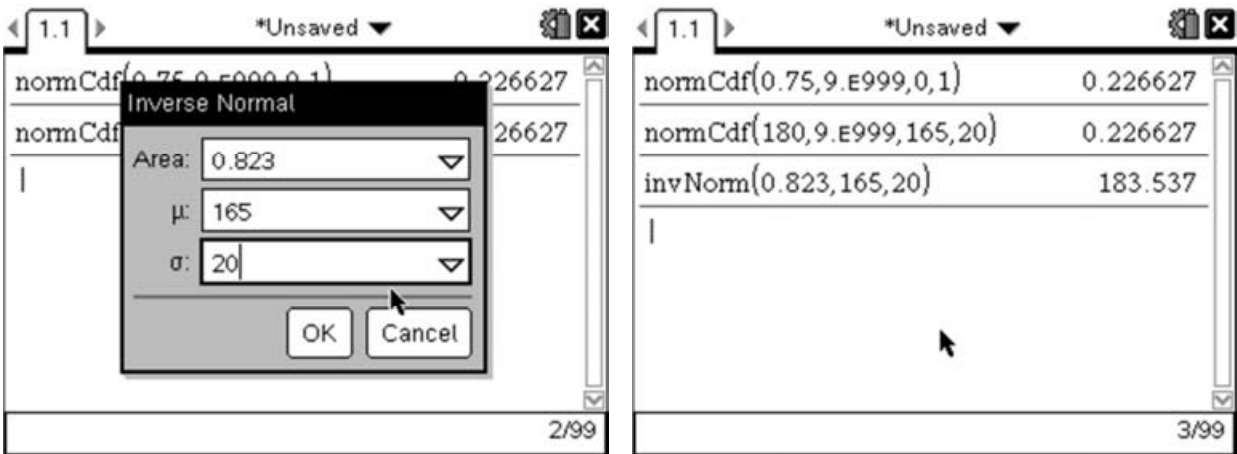
This gives us the value for the height as h= 183.53717015501… The value displayed on the screen is usually rounded to 6 significant figures if it is a decimal – integers values are always displayed exactly. To get it to display more places, simply move your cursor up over the answer and press (enter) to copy it down to the entry line where it will display all the available decimal places:
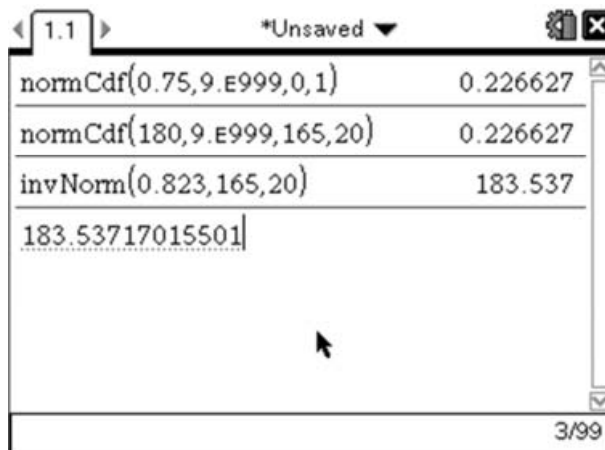
Markscheme:

$$P(X > 180) = P\left(Z > \frac{180-165}{20}\right) \qquad \text{(M1)}$$

$$P(Z > 0.75) = 0.227 \text{ (3sf)} \qquad \text{A1}$$

$$P(X < h) = 0.823$$

$$P\left(Z < \frac{h-165}{20}\right) = 0.823 \qquad \text{(M1)}$$

$$\frac{h-165}{20} = 0.9268\ldots \qquad \text{(M1)}$$

$$h = 184\text{cm (3sf)} \qquad \text{A1}$$

As all the method marks in this question are bracketed, full marks would be awarded for correct answers given to an appropriate degree of accuracy. It is important to note, whilst including some indication of your method is generally useful, the IB generally do not like to see answers which include calculator notation such as normCdf etc.

Binomial Distribution:

> **Question:**
>
> $X \sim B(7, p)$. Find the values of p if P($X = 3$) = 0.07. For each value of p calculate $P(X \geq 5)$
>
> [6 Marks]

Given that X is distributed Binomially we know the exact form of P(X=3) is given by:

$$P(X = 3) = {}^7C_3 \times p^3 \times (1 - p)^4$$
$$= 35 \times p^3 \times (p^4 - 4p^3 + 6p^2 - 4p + 1)$$
$$= 35(p^7 - 4p^6 + 6p^5 - 4p^4 + p^3)$$
$$= 35p^7 - 140p^6 + 210p^5 - 140p^4 + 35p^3$$

But the question tells us that this must be equal to 0.07, so we need to solve this polynomial of order 7 to find the value of p. The TI-Npsire has a function for finding the roots of polynomials which we can use in this case (in fact in general there is no nice method to find the exact solution of a polynomial of order 7, which follows as a result of a piece of Mathematics called Galois Theory).

To find the roots of a polynomial we must first put it into the required form P(x)=0. We have:
$$35p^7 - 140p^6 + 210p^5 - 140p^4 + 35p^3 = 0.07$$
Which we can rewrite to give:
$$35p^7 - 140p^6 + 210p^5 - 140p^4 + 35p^3 - 0.07 = 0$$

Like many of the functions on the TI-Nspire, we have the choice here of either typing the function using the letter keys or using the wizard which is available from the menu. I will outline both approaches.

Manual Approach:

The function we want is called polyRoots (there is a related function called cPolyRoots – which will return complex roots as well as real solutions). The syntax for this function is polyRoots(Polynomial,variable); so in our case what we want to enter is:
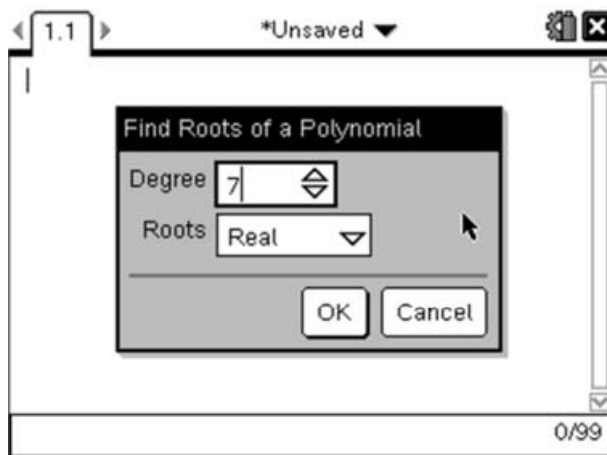
$$\text{polyRoots}(35p^7 - 140p^6 + 210p^5 - 140p^4 + 35p^3 - 0.07, p)$$

We can enter the polyRoots function by pressing (menu)(3)(2). Then enter the function as above and press (enter) to evaluate the function.
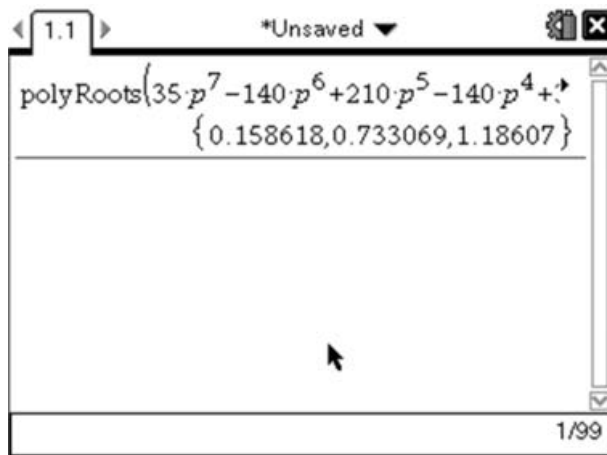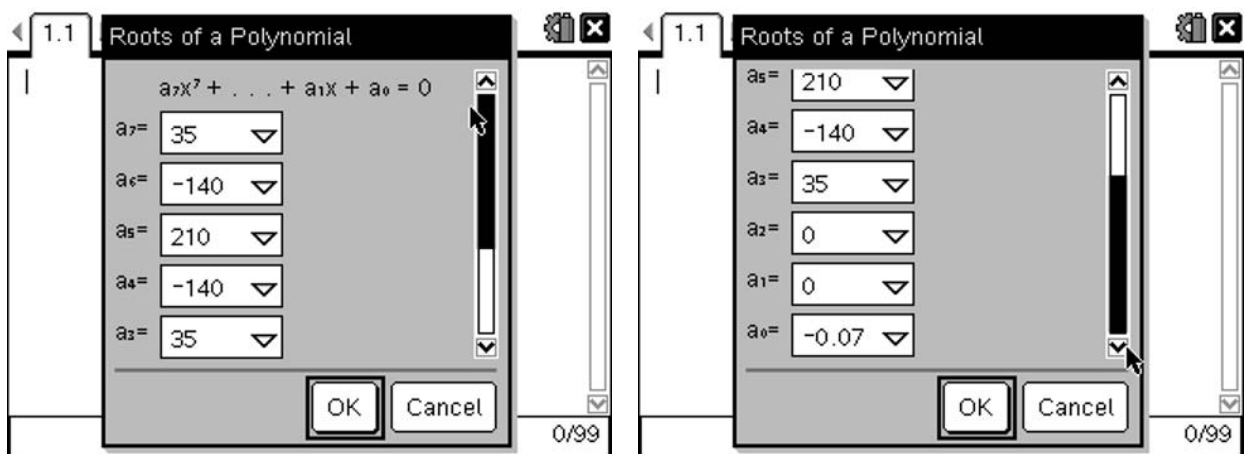
Using the Wizard:
Alternatively we can use the wizard to walk us through the process. To do this start the wizard by selecting (menu)(3)(3)(1), which will bring up the dialogue box:


Enter the degree of the polynomial (the highest power), and select whether you want Real roots or Complex roots, then select OK.

Enter the coefficients for each of the terms in the polynomial, remembering to include the sign and put zero for any terms that are missing. Then select OK.
This will enter the function on to the Calculator page. This gives you the opportunity to change it if necessary before pressing (enter) to evaluate the result.
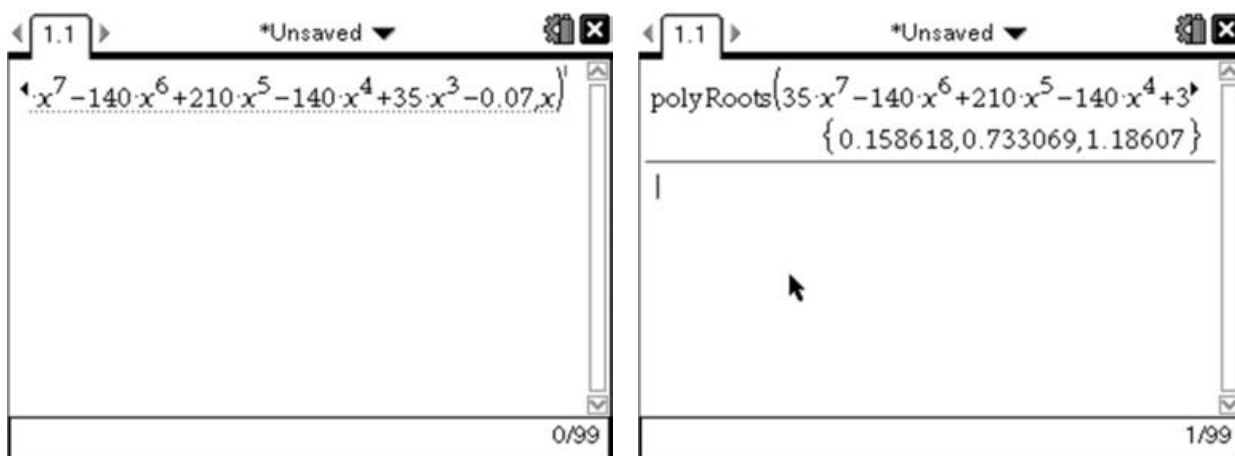
Note that both these methods give us three solutions:
- p=0.15861795444212
- p=0.73306913138531
- p=1.1860693868173

We can eliminate the third solution as we cannot have a probability greater than one! This leaves two possible valid solutions.

An alternative approach:
A completely different approach to this question would be to use the BinomPdf function combined with the nSolve function.

The nSolve function is a numerical equation solver which will return a single solution to an equation in a single variable. In our case, we want to use it solve a problem involving the Binomial distribution. The TI-Nspire has a function for evaluating the Binomial distribution (both at a particular value and across an interval). For this part of the problem we will need to use the BinomPdf function. The syntax of this function is binomPdf(number of trials, probability of success, x-value). So what we are given in the question can be expressed as:

$$binomPdf(7,p,3)=0.07$$

As we want to find a value for p, we can use this combined with the nSolve function as follows:

$$nSolve(binomPdf(7,p,3)=0.07,p,0)$$

The syntax here is nSolve(equation, variable, lowerbound), so what we have asked the TI-Nspire to do is find the first value of p greater than zero for which binomPdf(7,p,3)=0.07.

We can enter this on the TI-Nspire either by typing it out in full using the keyboard at the bottom (note the functions are not case sensitive), or by using a combination of the functions on the menu. To enter the nSolve function we can press (menu)(3)(1), then enter the BinomPdf function by pressing (menu)(5)(5)(D), which will start the wizard for this function.
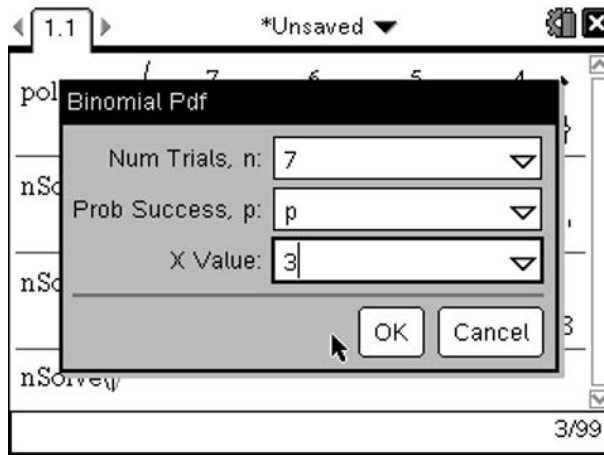
Now we enter the information we have; n=7, x=3. Enter the probability as 'p' which we will solve for. This will return an error message about insufficient arguments, as we have not yet entered the rest of the nSolve function. Press OK to continue and we can complete the rest of the nSolve function.
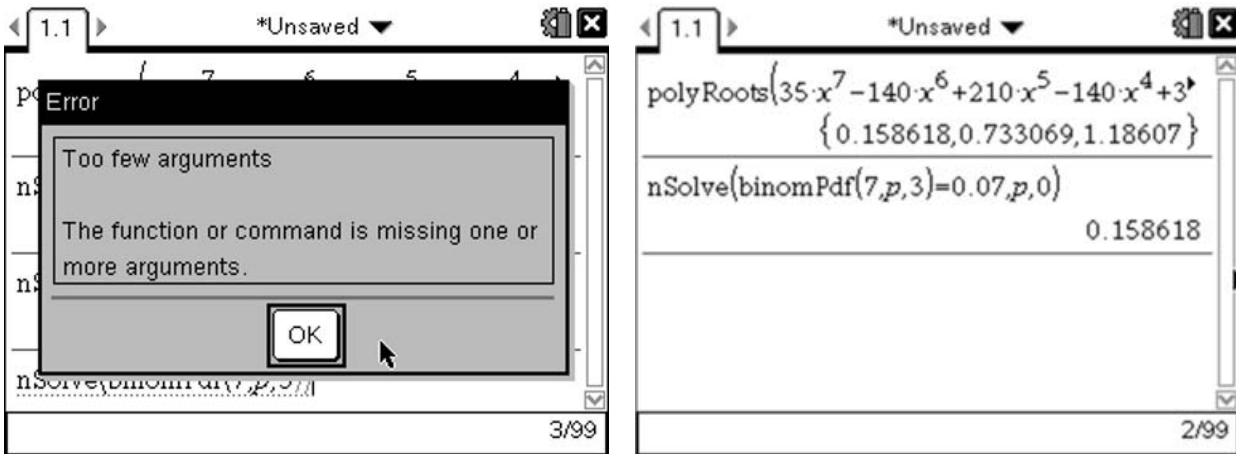
Notice this only returns the first of the solutions we found using the PolyRoots method, so it is always a good idea to see if there are any more solutions. We can do this by reusing the function we just entered but changing the lowerbound to be the solution we just found.

To do this press up twice on the navpad (▲) so that the previous calculation is highlighted and press (enter) to copy it down on to the entry line. We can now edit this expression by pressing 'delete' to remove the bracket and the 0. Finally press 'up' once on the navpad to highlight the answer to our previous calculation (i.e. 0.158618) and press (enter) to copy this down on to the entry line so that it now reads:

$$nSolve(binomPdf(7,p,3)=0.07,p,0.15861795444212)$$

Then press (enter). This will now look for the next solution to the equation greater than the first solution. If you suspect there might be a third solution you can repeat this process again changing the lower bound to be the second solution. If you do this for this example you will receive a domain error, as the third solution we found with the PolyRoots function is outside the domain of the binomPdf function.

$P(X \geq 5)$: The second part of this question asked us to find the probability of X taking a value greater than or equal to 5 for each of the values of p we found in the first part of the question. To answer this we can use the binomCdf function. The syntax of this function is binomCdf(n,p,lowerbound,upperbound). So to answer our question we want to enter:

$$binomCdf(7,p,5,7)$$

To make this easier, it is sensible for us to store the values we have found for p as variables. To do this we can enter p1:= 0.15861795444212. To do this press $\boxed{P}\boxed{1}$ ⓒtrl ⓜ(ⁱ) then press up to select the appropriate value from the previous calculations then press ⓔnter to copy the value down and finally press ⓔnter again to store the value as p1. Then repeat the process to store the second value as p2.
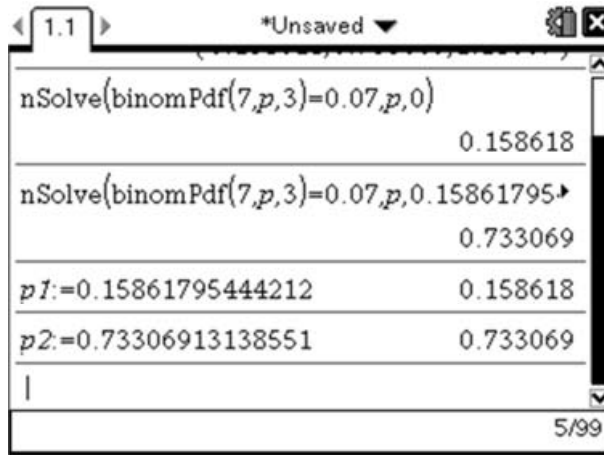
Now we can evaluate the expressions by pressing ⓜenu ⑤ ⑤ Ⓔ which will bring up the wizard for the Cumulative Binomial Probability function. Enter the values as shown below then select OK.
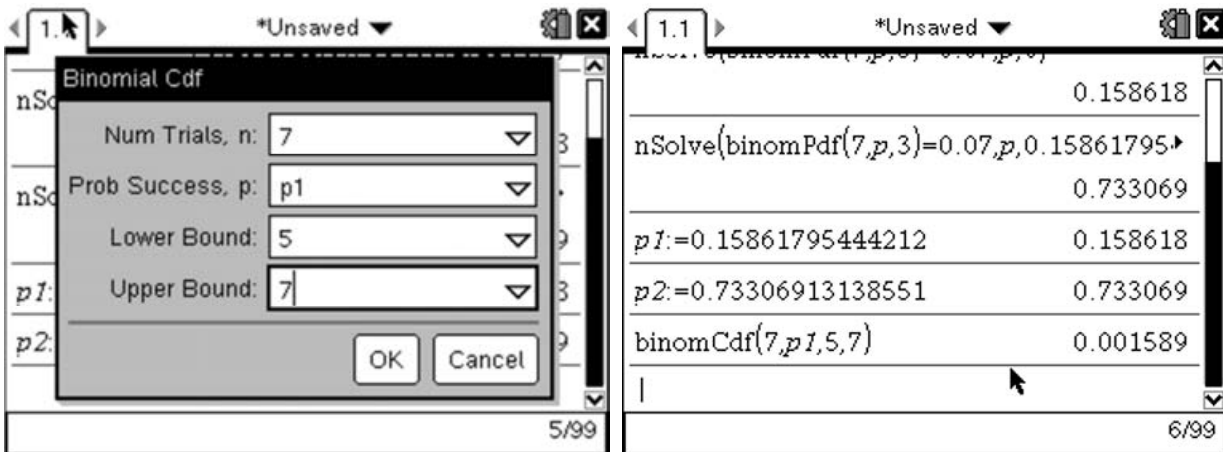
Then repeat this process for p2.
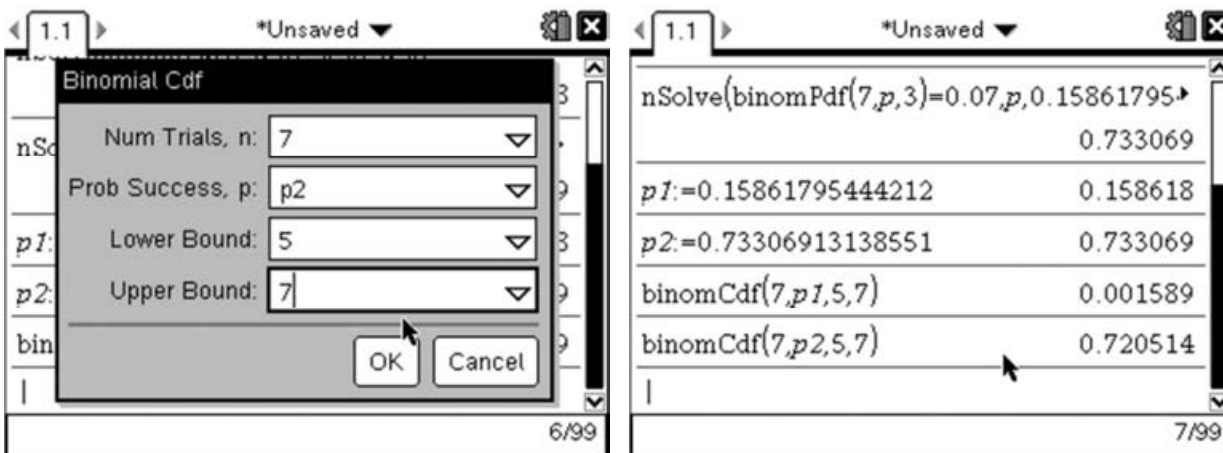
These two functions then give us the following results:

When p=0.159 (to 3sf) then $P(X \geq 5) = 0.00159$ (to 3sf)
When p=0.733 (to 3sf) then $P(X \geq 5) = 0.721$ (to 3sf)

Markscheme:

$^7C_3 \times p^3 \times (1-p)^4 = 0.07$ (M1)

$35p^7 - 140p^6 + 210p^5 - 140p^4 + 35p^3 - 0.07 = 0$ (A1)

p=0.15861795444212   A1

p=0.73306913138531   A1

p=1.1860693868173 (reject as p>1)   R1

p=0.159 then $P(X \geq 5) = 0.00159$ (to 3sf)

p=0.733 then $P(X \geq 5) = 0.721$ (to 3sf)   A1

Notice in the question the method marks for finding the three possible values for p are implicit. However you must explicitly state the p=1.186… values and then reject it (as probabilities can't be greater than 1!) to ensure you get full marks.

Numerical Integrals:

> **Question:**
> Find the value of a such that $\int_0^a \sin^2(x)dx = 1.234$ Give your answer to 3 decimal places.
>
> [3 Marks]

This questions can be solved completely on the TI-Nspire, gaining full marks. There are three approaches to solving this question on the TI-Nspire:

Semi-Analytical Approach:
In this approach we would begin by carrying out as much of the integral as we can by hand:

$$\int_0^a \sin^2(x)dx = \int_0^a 1 - \cos^2 x \, dx = \int_0^a \frac{1}{2} - \frac{1}{2}\cos 2x \, dx = \left[\frac{x}{2} + \frac{1}{4}\sin 2x\right]_0^a$$

$$\Rightarrow \left[\frac{x}{2} + \frac{1}{4}\sin 2x\right]_0^a = \frac{a}{2} + \frac{1}{4}sin2a = 1.234$$

This equation then needs to be solved using one of the two methods outlined below.
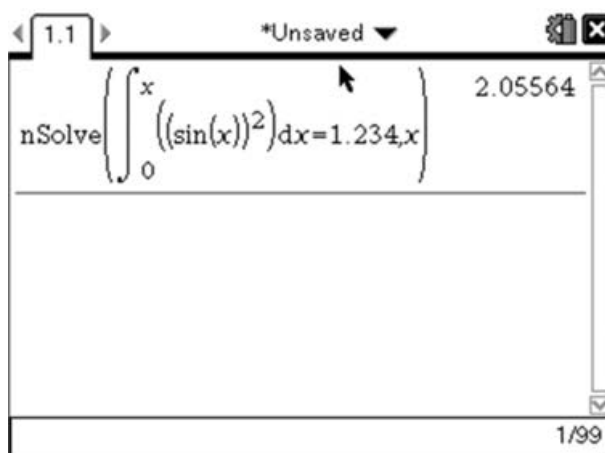
nSolve() method:
The nSolve() function can be used to find a numerical solution to an equation. The general syntax is nSolve(equation, variable). So to solve our equation we would do:

$$nsolve\left(\int_0^x \sin^2(x)dx = 1.234, x\right)$$

Note: On the TI-Nspire we cannot use different variables for the limits to the function, so the upper limit must also be called x. This is a limitation of working on a non-CAS device.

To enter this on the handheld, first create a Calculator page, then press (menu)(3)(1) to insert the nSolve function. Press (≡) then select the numerical integral template. Enter the limits and function followed by the variable you are integrating with respect to, then (,)(X), then finally (enter):

If we needed a solution within an interval such as $1 \leq a \leq 4$, then we can add limits to our nSolve like:

$$nsolve\left(\int_0^x \sin^2(x)dx = 1.234, x, 1, 4\right)$$

Graphical Method:

The graphical method uses a similar approach to the nSolve() method, but instead works by graphing the function:

$$f1(x) = \int_0^x \sin^2(x)dx$$

To do this, add in a Graphs page and enter the function as follows:

Use the Numerical Integral template (Press ⓜⓐⓣⓗ and select the Numerical Integral template on the bottom row). Enter the limits as ⓪ and Ⓧ, then enter the function by pressing ⓘ ⓣⓡⓘⓖ ⓔⓝⓣⓔⓡ Ⓧ ⓘ ⓧ² then move right and enter the variable you're integrating with respect to – in our case Ⓧ, then press ⓔⓝⓣⓔⓡ
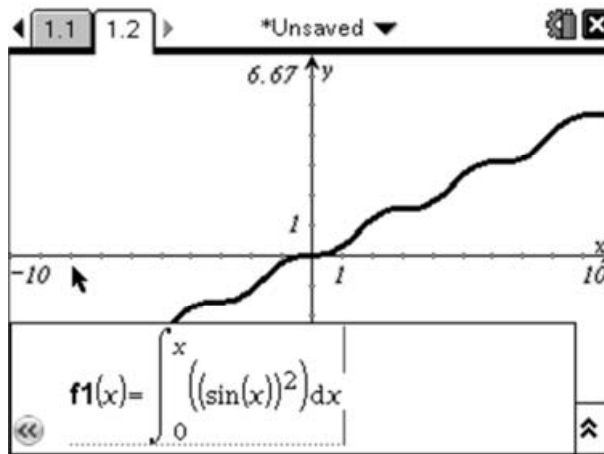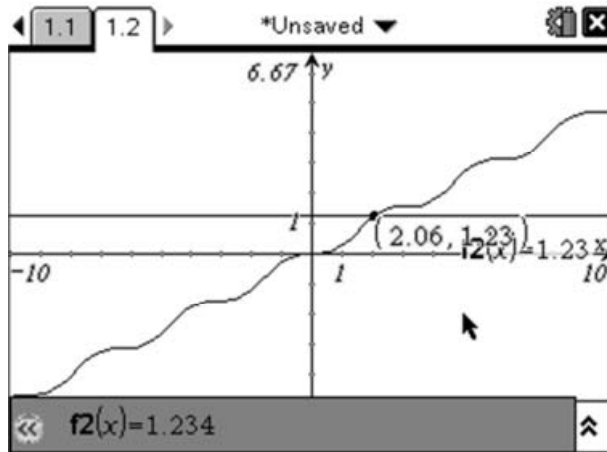

Now draw the line y=1.234, by entering a second function:

$$f2(x) = 1.234$$

Press ⓣⓐⓑ or ⓒⓣⓡⓛ Ⓖ to bring the function entry line back up.
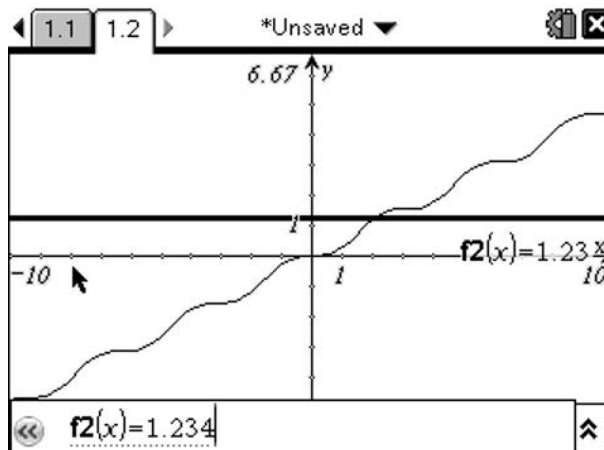
To find the intersection of these two curves, and therefore solve the original question, we need to simply use the 'intersection point(s)' tool. To do this press (menu)(7)(3), then select the curve and the straight line.

To increase the accuracy of the point, you can right click ((ctrl)(menu)) on the relevant co-ordinate – in this case 2.06, then select 'Attributes':
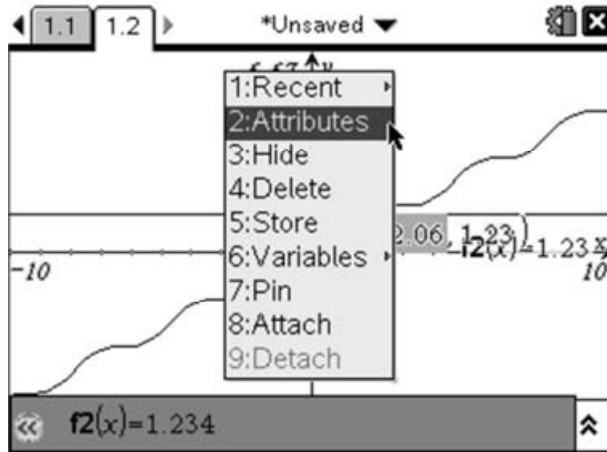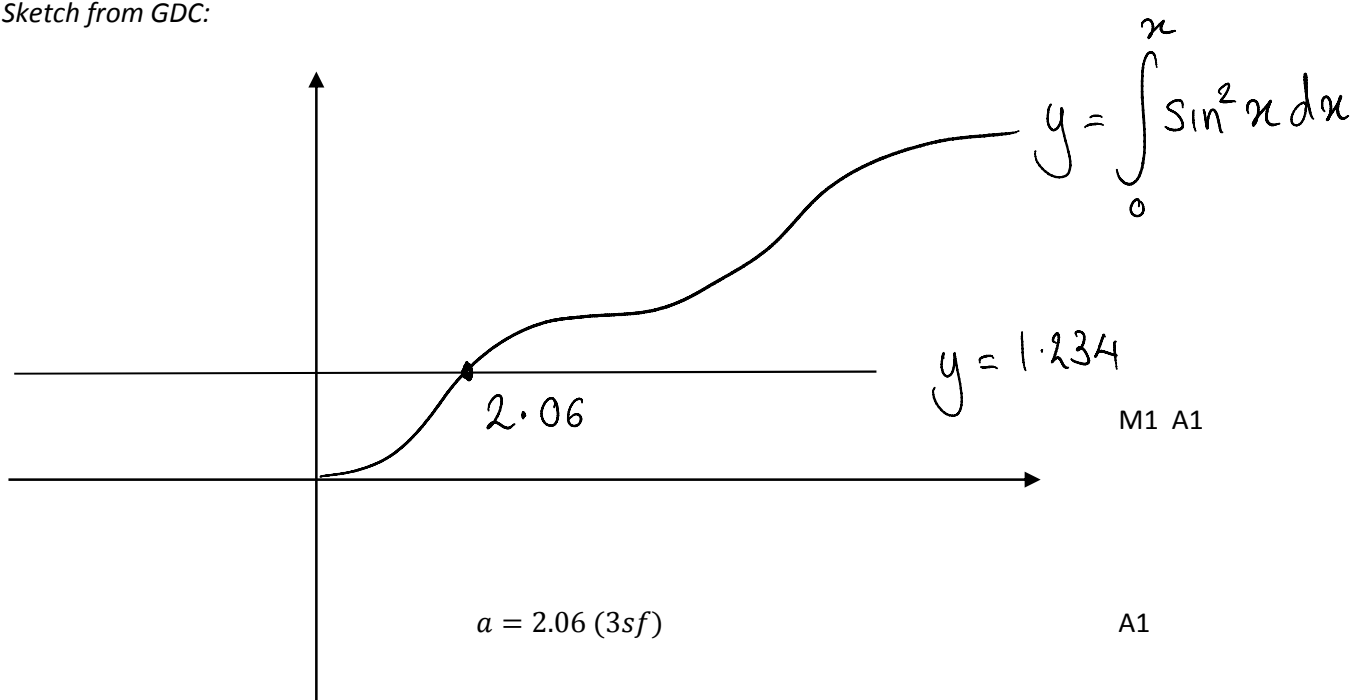

Then select the number of decimal places you want displayed:

Markscheme:

Sketch from GDC:

$$y = \int_0^x \sin^2 x \, dx$$

$$y = 1.234$$

M1 A1

2.06

$a = 2.06 \ (3sf)$

A1
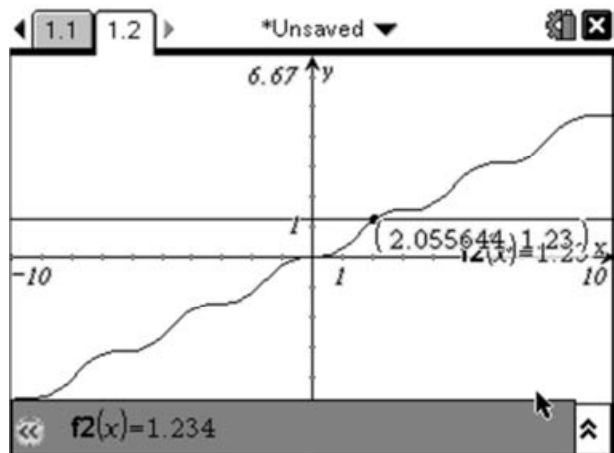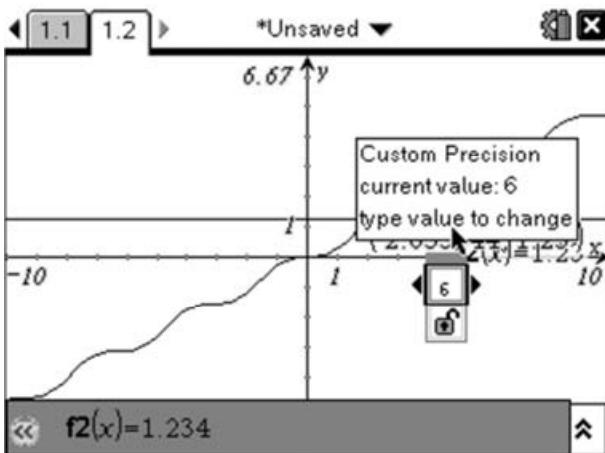
As with all questions which are solved using the GDC, it is advisable to include a sketch copy of any graphs used to explain to the marker how you approached the questions

Modulus Inequalities:

Question:
Find the set of values for which:

$$\left| \frac{x^2 + 10x}{x^2 - 10} \right| \le 2$$

[6 Marks]

Whilst it is possible to handle modulus function inequalities analytically, it is usually a time consuming process.

In this example a graphical approach is the quickest way of tackling the problem. To do this we must first insert a Graphs page ((ctrl)(doc▾) then select Add Graphs). On this new Graphs page, we will draw two functions; the first will be the left-hand side of the inequality and the second will be the right-hand side.

To enter the left-side as f1(x) we will make use of several of the templates:

The first template we need is for the modulus function that surrounds the left-hand side. We can get the modulus template by pressing (⊞) and then selecting the modulus template. Then move the cursor inside the modulus function and insert a fraction template either by pressing (⊞) and selecting the appropriate template or by pressing (ctrl)(÷). Finally, enter the function as shown above using the (X) key from the bottom for the x variable. This should give you:
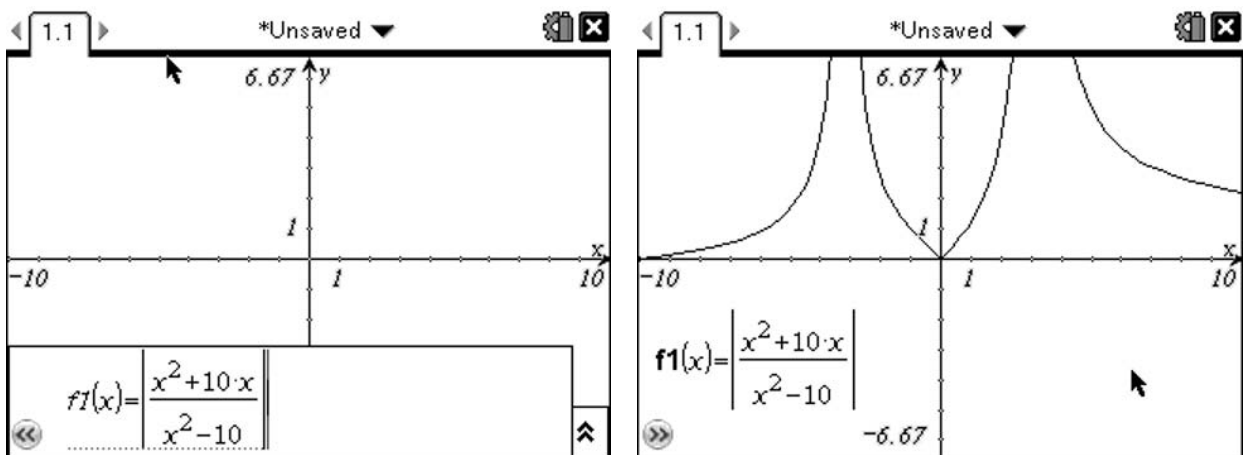

Next we need to insert the equation which represents the right-hand side of this inequality. In this case it is just the function f2(x)=2:
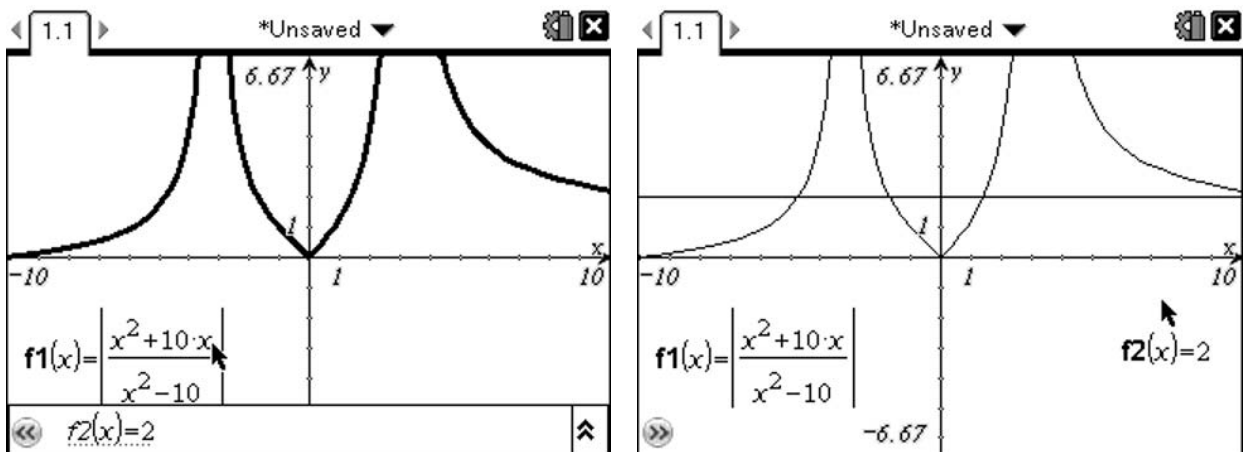
Looking at the graph we can see three intersections. However, it looks like there might be another solution to the right of the screen, so we should either zoom out or pan the screen left. You can zoom out in various ways. One option is to use the zoom and windows settings available from the Menu ((menu)(4)). The other alternative is to grab (hold (✣) for a few seconds or press (ctrl)(✣)) on to one of the tick marks on an axis then drag it toward the centre to zoom in or towards the edge to zoom out. Holding down (⇧shift) whilst doing this will zoom in only one axis.

Alternatively, you can pan by grabbing any empty space on the page, then dragging the page to the left to show the solution that is off the page.
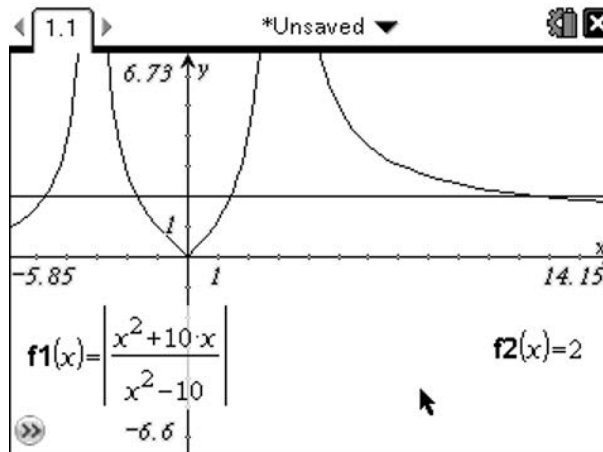

Now we need to find the intersection points of the curve with the straight-line. We can do this using the 'intersection points' tool ((menu)(7)(3)) then select the curve followed by the straight line. This will insert points at the intersections.
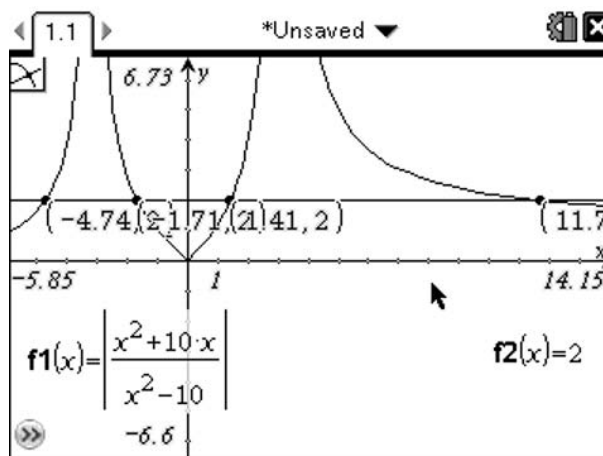
As there are a lot of points together, the co-ordinate labels have overlapped. We can separate them by grabbing the labels and moving them to more convenient places on the page.
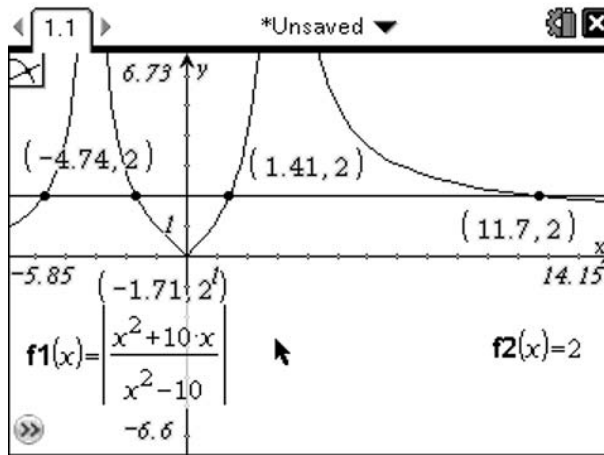
Usually 3 significant figures is sufficient for a question of this sort, but if we needed more we could select the relevant co-ordinate by moving the cursor over the co-ordinate then right-clicking (ctrl menu) and selecting the Attributes option. This then gives you a dialogue box which can be used to change the precision of a number.
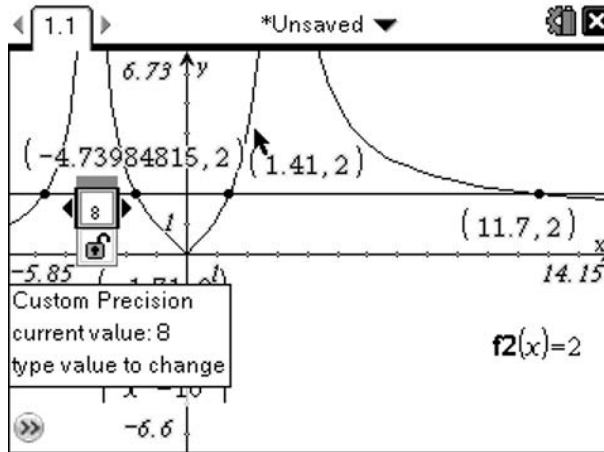
The final step is to look back at the original question and work out which sides of the points we are interested in. The original question said:

$$\left| \frac{x^2 + 10x}{x^2 - 10} \right| \leq 2$$

So we are looking for those regions where the curve intersects, or is below the straight-line:
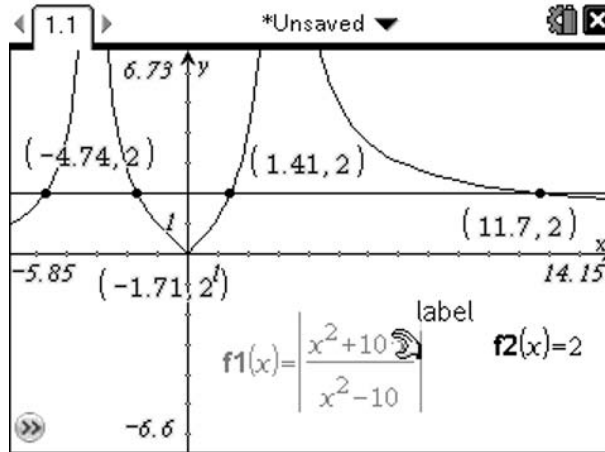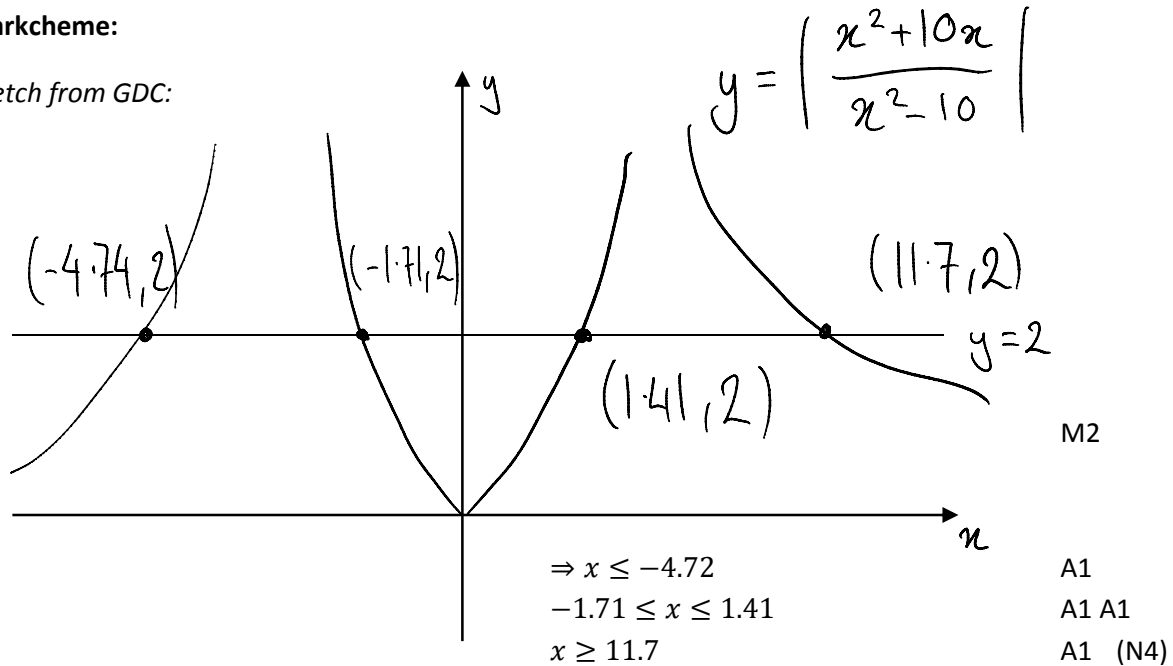

So we are interested in all points to the left of the first point, those between the second and third point, or those to the right of the fourth point. In terms of intervals we would normally write these as:

$$x \le -4.72 \text{ or } -1.71 \le x \le 1.41 \text{ or } x \ge 11.7$$

Markcheme:

Sketch from GDC:

$$\Rightarrow x \le -4.72 \qquad\qquad \text{A1}$$
$$-1.71 \le x \le 1.41 \qquad\qquad \text{A1 A1}$$
$$x \ge 11.7 \qquad\qquad \text{A1 (N4)}$$

For this question you would only receive 4 out of the 6 marks (N4) for writing down the correct answer without including a sketch of the graph. It is important to include a sketch graph whenever you include answers based on a graph from the GDC.

Working with Vectors:

> **Question:**
> Given that $v = 2i + 3j - k$ and $w = i - j + k.$ Find
> i) the angle in radians between the two vectors
> ii) $v \times w$
>
> [4 Marks]

The standard method for finding the angle between two vectors comes from the definition of the dot product of two vectors:

$$v \cdot w = |v||w| \cos \theta$$

Which can be rearranged to give us:

$$\cos \theta = \frac{v \cdot w}{|v||w|}$$

The TI-Nspire has several functions which are of use when working with vectors. For this question we need to perform a dot product and find the size of a vector. To make it easier for us to work with the vectors in this case, we will begin by defining the vectors. We can do this by pressing V⃝ctrl⃝ which gives us 'v:=' we now need to enter a vector template. We can do this by pressing ⃝ and selecting the 2D vector template

$\begin{bmatrix} \square \\ \square \end{bmatrix}$
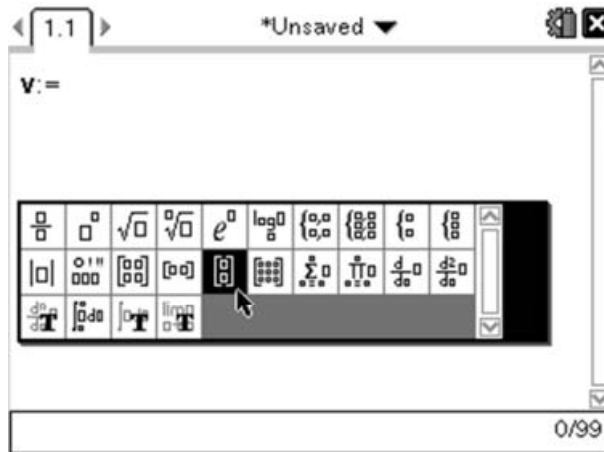

This will give us a 2D vector. However, we need a 3D vector, so we should enter the first two values. Then press ⏎ (bottom right corner) which will add another cell to the vector, in which we can enter the value for the **k** component of the vector.
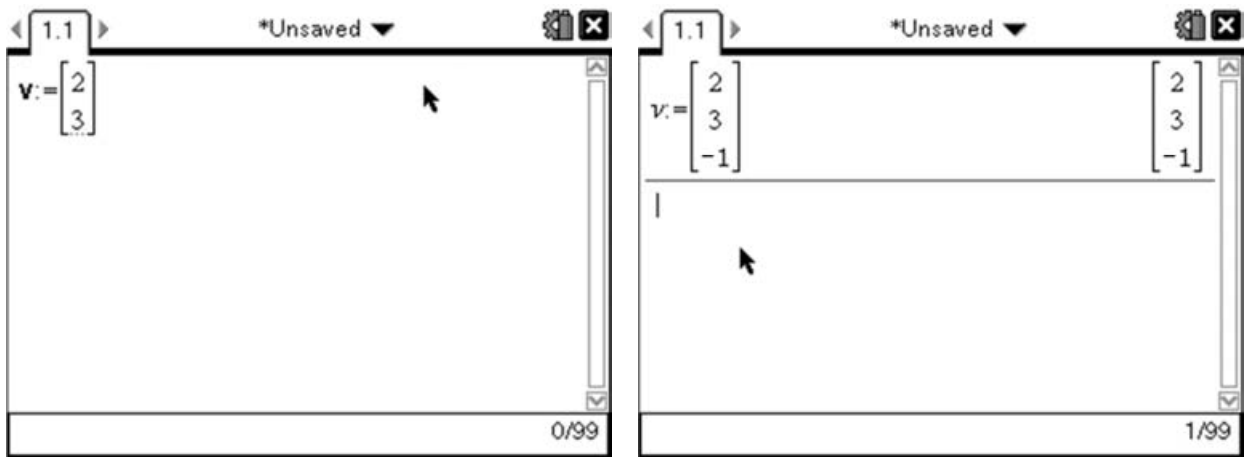
The alternative method is to select the general matrix template $\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$ and then select 1 column and 3 rows:
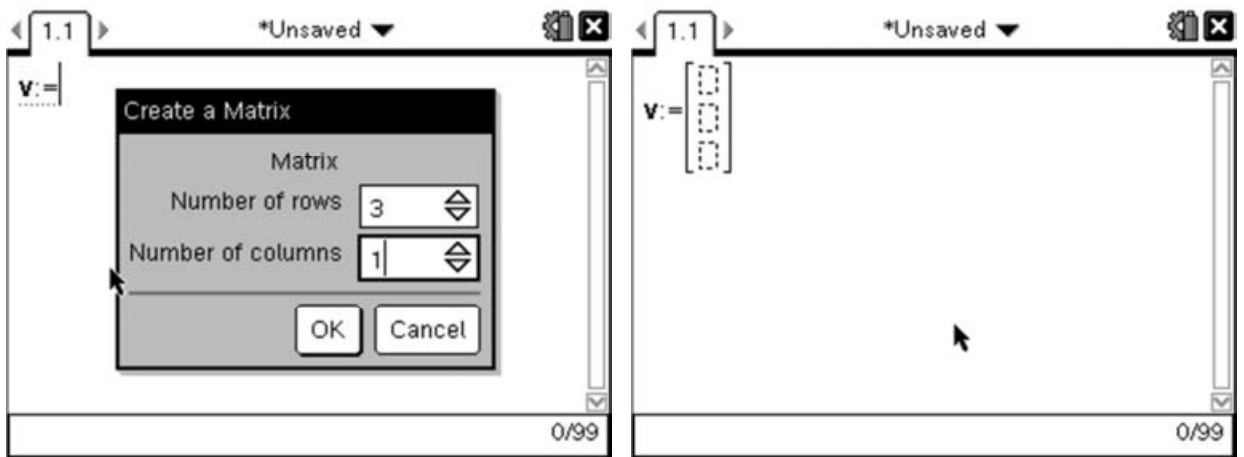
Repeat this process to define the vector **w**:
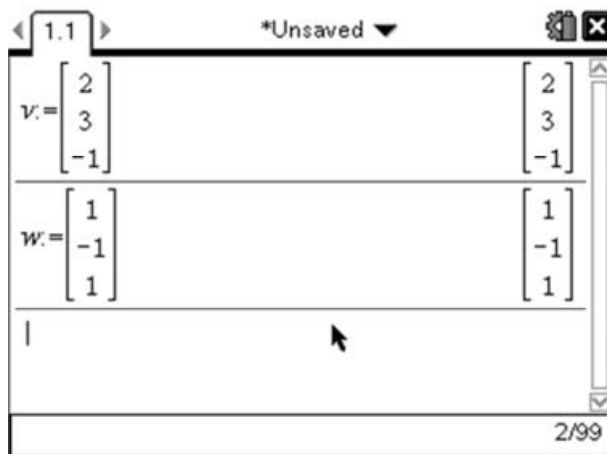
Now we need a way of finding the dot product and size of these vectors. Again the TI-Nspire has functions to perform these calculations. To find the dot product of two vectors we need to use the DotP (menu 7 C 3) function of **v, w.**

To find the size of a vector we use the 'norm' function such that $norm(v) = \sqrt{2^2 + 3^2 + (-1)^2}$.

Looking back at our original work, we can see that what we need to find is:

$$\theta = \cos^{-1}\frac{\boldsymbol{v} \cdot \boldsymbol{w}}{|\boldsymbol{v}||\boldsymbol{w}|}$$

We can enter this into the calculator by pressing (trig) and selecting $\cos^{-1}$. Next enter a fraction template (ctrl)(÷). In the top of the fraction press (menu)(7)(3) to enter the DotP function, then enter **v**, **w**. On the bottom of the fraction, you can then enter the norm function by pressing (menu)(7)(7)(1), then entering the appropriate vector to give:
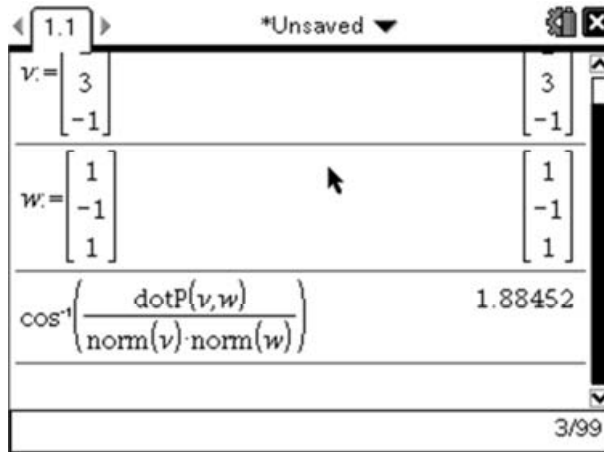
Which returns the angle between the two vectors expressed in radians. If we wanted to convert this value to degrees we could do this by pressing the Catalogue button (which gives access to all the functions), then scroll through till you find ▶DD and press (enter).
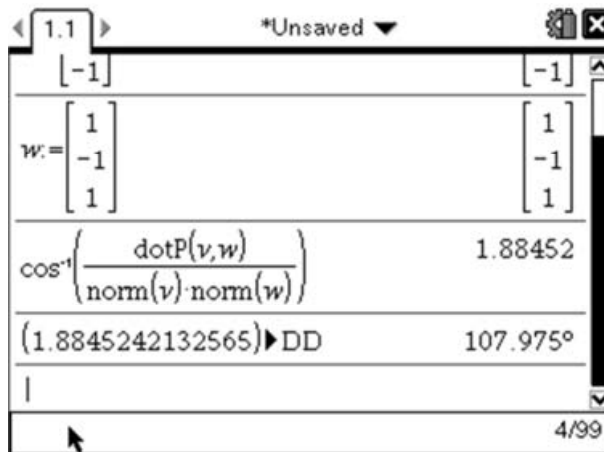

For the second part of the question, we are asked to find the cross product of the vectors **v** and **w**. The traditional method for finding cross products of 3D vectors is by finding the determinant of the following 3x3 matrix:

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

In this particular example this means finding:

$$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 3 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \boldsymbol{i}\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} - \boldsymbol{j}\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + \boldsymbol{k}\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 2\boldsymbol{i} - 3\boldsymbol{j} - 5\boldsymbol{k}$$

However on the TI-Nspire we have another function similar to DotP, called CrossP which can be used to find the cross product of two vectors. We can use the CrossP function by pressing (menu)(7)(C)(2). Then, as our vectors are already defined, we can simply put CrossP(v,w):
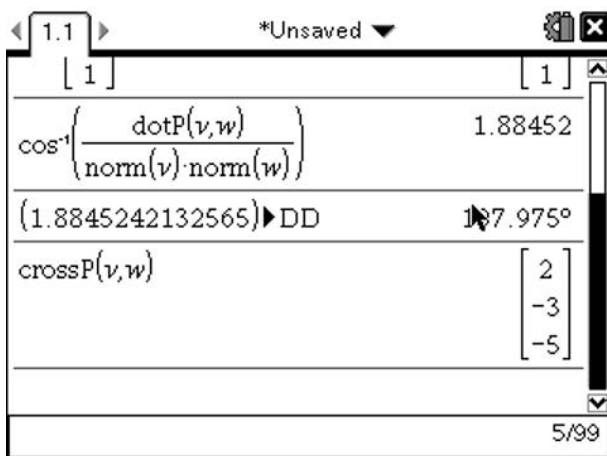
Markscheme:

$\cos\theta = \frac{v \cdot w}{|v||w|}$

$\cos\theta = \frac{\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{2^2+3^2+(-1)^2} \times \sqrt{1^2+(-1)^2+1^2}}$ (M1)

$\cos\theta = \frac{2-3-1}{\sqrt{14}\sqrt{3}}$

$\cos\theta = \frac{-2}{\sqrt{42}}$

$\theta \approx 1.88^r$ A1

$\begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 1 & -1 & 1 \end{vmatrix}$

$= i\begin{vmatrix} 3 & -1 \\ -1 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$ (M1)

$= 2i - 3j - 5k$ A1

Notice that in this question the method is implicit so full marks are awarded for stating the solutions.

Inverse Matrices and Simultaneous Equations:

> **Question:**
> Write down the inverse of the following matrix:
> $$\begin{pmatrix} 1 & 3 & -5 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$
>
> Hence solve the following system of simultaneous equations:
> $$\begin{cases} x + 3y - 5z = 11 \\ 3x - 2y + z = 3 \\ 2x + y - 2z = 4 \end{cases}$$
>
> [4 Marks]

Finding the inverse of a 3x3 matrix is a long and time consuming process. Thankfully the TI-Nspire can do this for us. Entering and working with matrices on the TI-Nspire is a simple process.

The first step to working with Matrices is to insert a Calculator page (ctrl doc▾ then select Add Calculator). We can enter matrices of almost any size (up to 99x99) by using the matrix template. This template can be accessed using the template key and then moving the cursor down until you reach the option that looks like a 3x3 matrix and pressing (enter). This will present you with a dialogue box to choose the size of your matrix, but since this defaults to 3x3. We can simply press (enter) again to create the template.
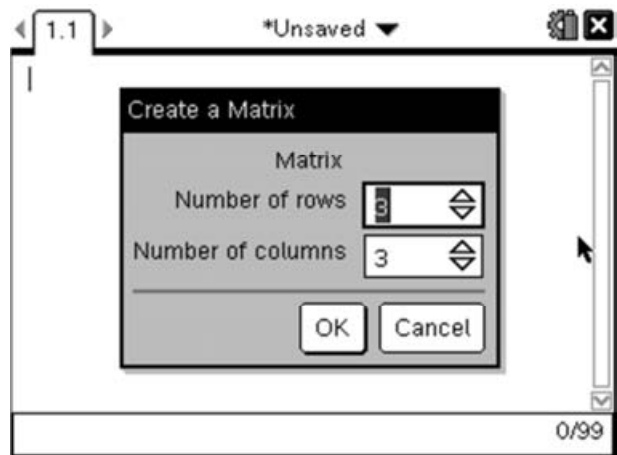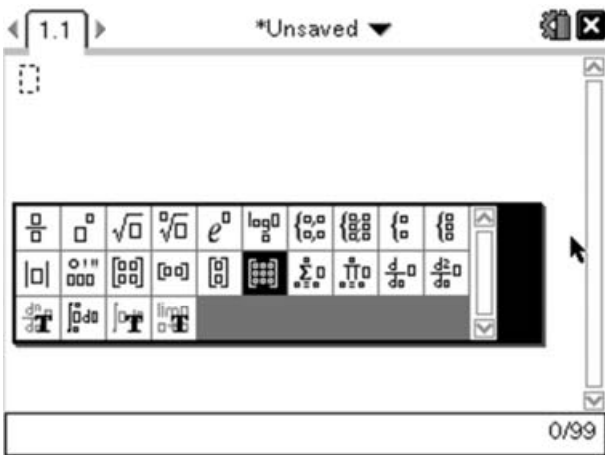
Then simply move around within the matrix by clicking up, down, left and right on the touchpad and enter the appropriate values in each cell:
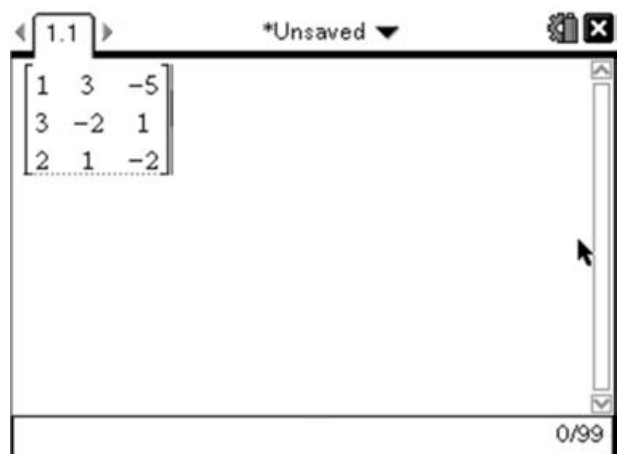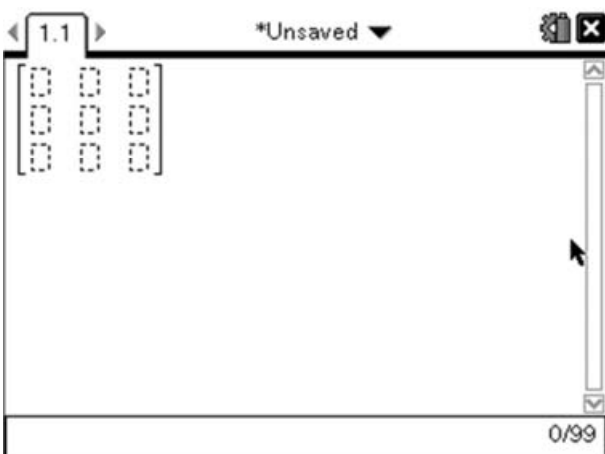
We want the inverse matrix. If this matrix was called A, then we would usually label the inverse matrix as A^{-1}, and this is exactly what we do on the TI-Nspire. To do this, we use the 'to the power of' key ($\wedge$) and enter the value -1. Pressing (enter) will then return the inverse matrix:
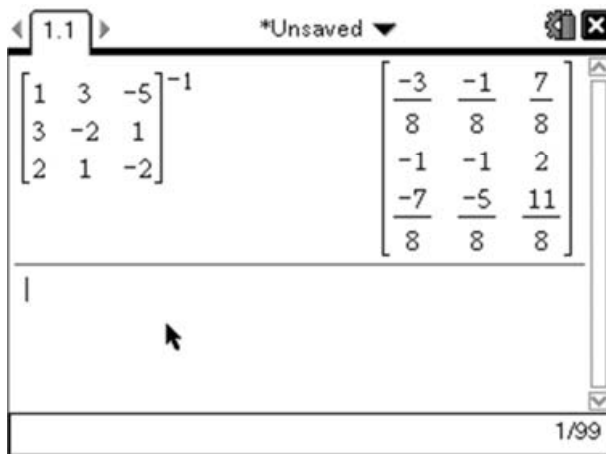
Given that the majority of the matrix (excluding the middle row) are fractions with 8's in the denominator, we may prefer to take the $\frac{1}{8}$th out of the Matrix. We can do that by simply multiplying the resultant matrix by 8. To do this press (8)($\times$) then press 'up' to highlight the answer from the previous calculation and press (enter) to copy it down and (enter) again to carry out the calculation.
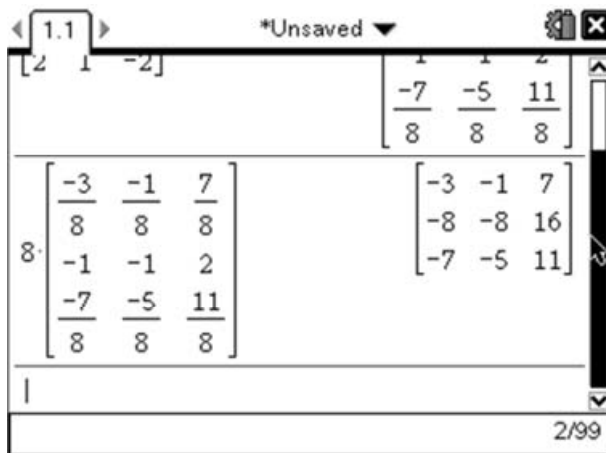
So we can say that the inverse matrix is:

$$\frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}$$

The second part of the question asks us to **hence** solve the simultaneous equation. If a question uses the word hence, then your solution to the next part must use your answer to the previous part. In this case we are asked to use the inverse matrix we have just found to help us solve the following system of equations.

$$\begin{cases} x + 3y - 5z = 11 \\ 3x - 2y + z = 3 \\ 2x + y - 2z = 4 \end{cases}$$

The first thing we have to do is write this in matrix form:

$$\begin{pmatrix} 1 & 3 & -5 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$

At this stage, hopefully you will recognise that the matrix in this equation is the same as the one from the first part of the question. We want to solve this matrix equation to find x,y and z, which we can do by left multiplying both sides of the equation by the inverse matrix:

$$\frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}\begin{pmatrix} 1 & 3 & -5 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$

But multiplying a matrix by its inverse gives us the identity matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$

And $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ so we are left with:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$

As we already know the inverse matrix from the first part of the question, and it is still stored on our handheld, we simply need to multiply the inverse matrix by the matrix $\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$ to obtain the values for x, y and z.

To do this press 'up' until the inverse matrix is selected, then press (enter) to copy it down on to the entry line. Then press (菜) and again select the symbol that looks like the 3x3 matrix. This time in the dialogue box enter 3 rows but just 1 column (press (tab) to move between the options) then press (enter). This will give you a 3D matrix.

Enter the values $\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$ in the matrix then press (enter).
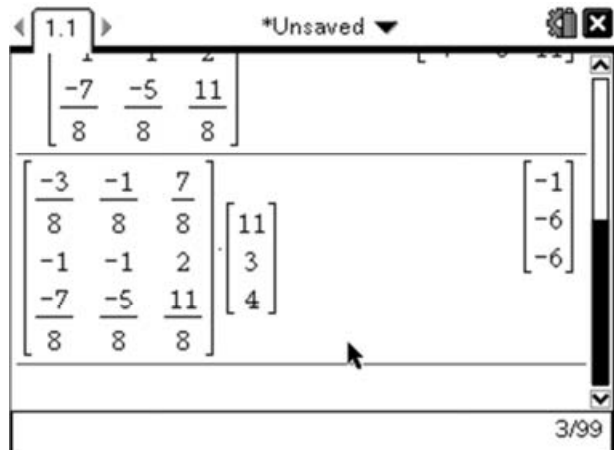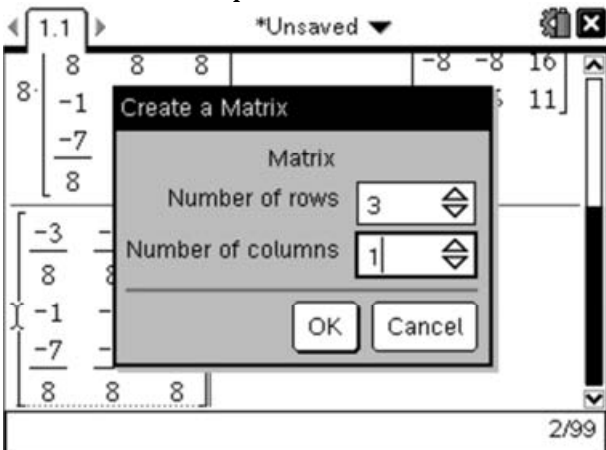
This tells us that the solution is: x=-1   y=-6 and z=-6.

We can use the built-in simultaneous equation solver to check this solution by pressing (menu)(3)(2) which will give you the simultaneous equation wizard. Change the 'Number of equations' to be 3 and press (enter):
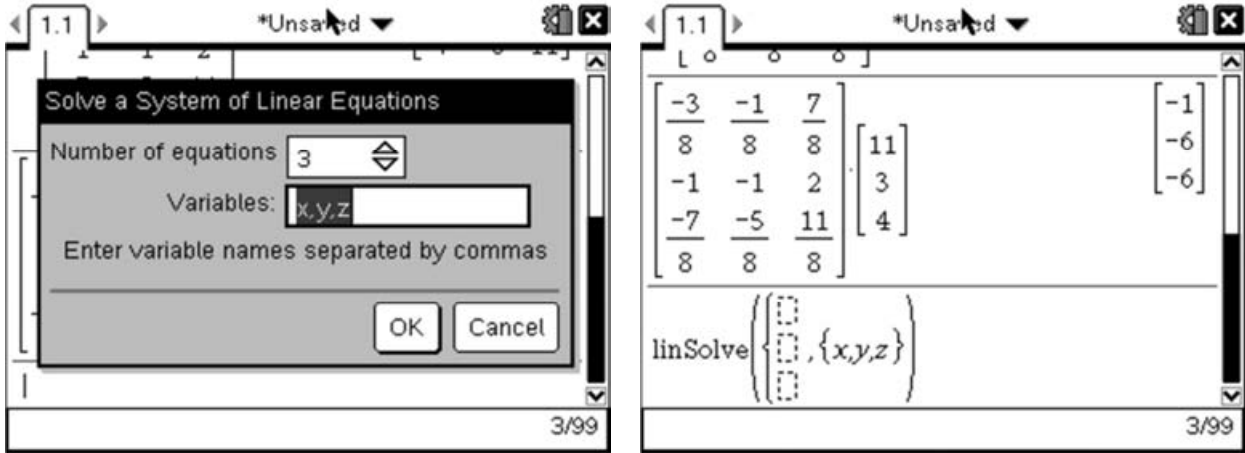
This gives us the template for solving the system of linear simultaneous equations. We just need to enter the three equations on the three lines of the template exactly as they were given in the question:
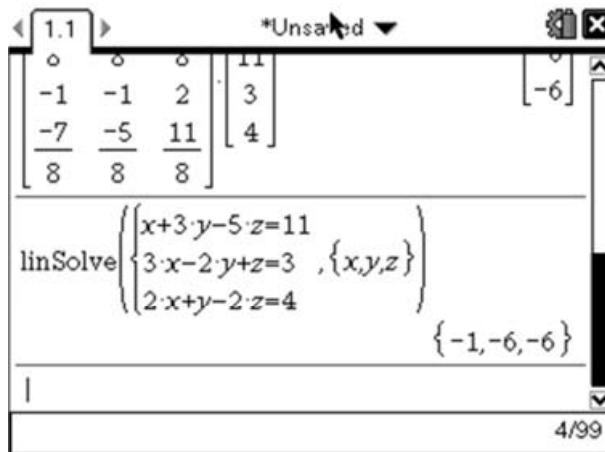
Pressing (enter) will then return the solutions, which match those we found using the matrix method. This should reassure us that we didn't make a mistake!

Aside: It is also worth considering how we deal with systems of equations which do not have a unique solution. Let's for example consider the following system:

$$\begin{cases} x + 5y - 3z = 13 \\ x + 2y + z = 4 \\ 2x + 7y - 2z = 17 \end{cases}$$

We can check that this system has no unique solutions by calculating the determinant of the matrix representation, which will come out as 0.
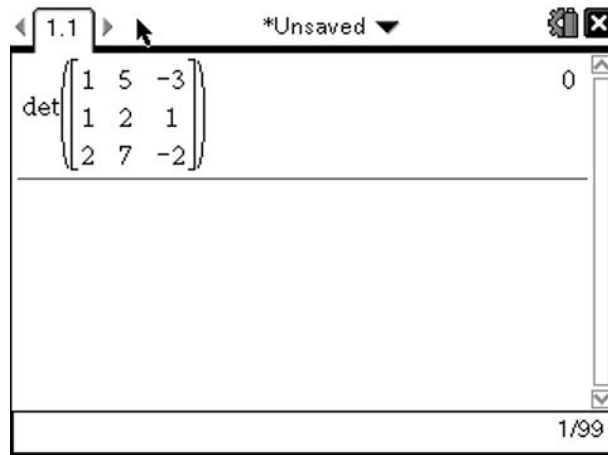
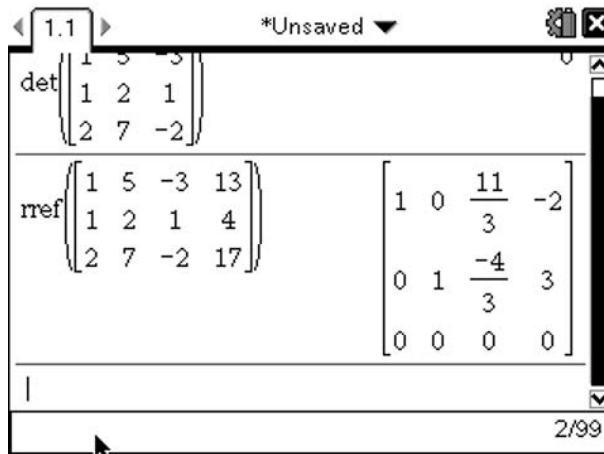The typical method to find the solution set is to consider the augmented matrix (i.e. the above matrix with an additional column for the constants) and then calculate the reduced row echelon form.

This is then interpreted as follows; the first three columns give the coefficients of 'x,y' and 2 respectively and the last column is the constant. So for this example the solution set is:

$$x + \frac{11}{3}z = -2 \quad \text{and} \quad y - \frac{4}{3}z = 3$$

This can be rearranged to give the following form:

$$z = \frac{-3(x+2)}{11} = \frac{3(y-3)}{4}$$

We can again use the simultaneous equation solver to check our solution set:
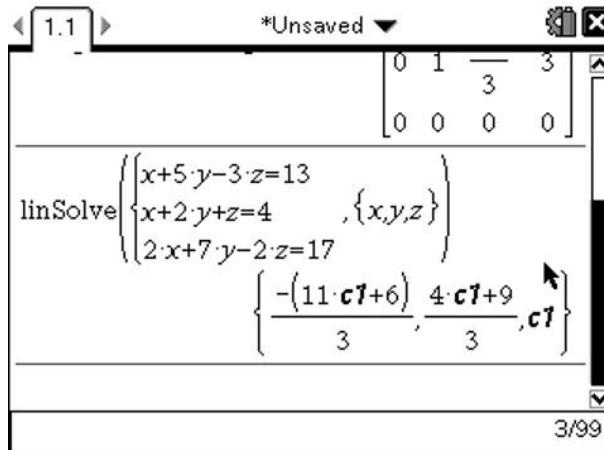
This solution takes a little interpretation, but we can see that each of the x, y and z values are given in terms of an unknown constant, in this case called **c1**. But as $z = c1$, we have that:

$$x = -\frac{11z+6}{3} \quad \text{and that} \quad y = \frac{4z+9}{3}$$

which are rearrangements of the solutions found above.

Markscheme:

$$A = \begin{pmatrix} 1 & 3 & -5 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}$$

$A^{-1} = \frac{1}{8}\begin{pmatrix} -3 & -1 & 7 \\ -8 & -8 & 16 \\ -7 & -5 & 11 \end{pmatrix}$ or equivalent (including decimal representation to 3sf)   A1

$$\begin{pmatrix} 1 & 3 & -5 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$ (M1)

$$A^{-1} A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1}\begin{pmatrix} 11 \\ 3 \\ 4 \end{pmatrix}$$ M1

$$I\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \\ -6 \end{pmatrix}$$

$\Rightarrow$ the three planes intersect at the point $(-1, -6, -6)$ A1

Notice how the markscheme awards specific marks for stating the matrix form of the equation, and for showing that you understand that the solutions to the system is given by multiplying the inverse matrix by the vector containing the constant terms. Without showing these steps you are likely to looks at least two marks on this question.

Complex Products:

> **Question:**
>
> Let z_1 and z_2 be complex numbers. Solve the following system of equations:
> $$2z_1 + z_2 = 7 + i$$
> $$iz_1 - z_2 = -3 + 6i$$
>
> Give your answer in the form $z = a + ib$, where $a, b \in \mathbb{Z}$
>
> [6 Marks]

There are several approaches we can use to tackle a problem of this form. The standard method is to use the 'elimination' method that would be used to solve a simple linear system:

$$2z_1 + z_2 = 7 + i \quad (A)$$
$$iz_1 - z_2 = -3 + 6i \quad (B)$$

Adding together these two equations eliminates z_2. Giving us:

$$2z_1 + iz_1 = 4 + 7i \quad (A) + (B)$$

Factorising the left hand side gives us:
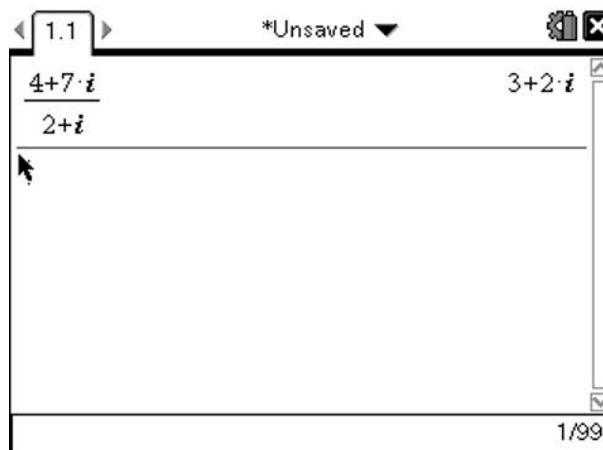
$$(2 + i)z_1 = 4 + 7i$$

So we have that:

$$z_1 = \frac{4 + 7i}{2 + i}$$

Division of complex numbers is usually carried out by multiplying the numerator and denominator by the complex conjugate:

$$z_1 = \frac{4 + 7i}{2 + i} \times \frac{2 - i}{2 - i} = \frac{8 + 14i - 4i - 7i^2}{4 + 2i - 2i - i^2} = \frac{15 + 10i}{5} = 3 + 2i$$

Alternatively the TI-Nspire can do the division:

To enter the imaginary number symbol press the $\textcircled{\pi}$ button to bring up the mathematical symbol options and select the i. Once we have the value of z_1 we can then substitute this into either of the original equations to find the value of z_2.
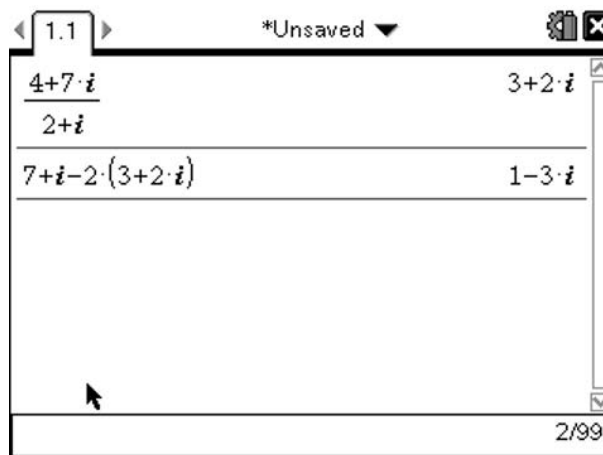
$$2z_1 + z_2 = 7 + i \quad (A)$$

Substituting in our value for z_1 gives:

$$2(3 + 2i) + z_2 = 7 + i$$

$$6 + 4i + z_2 = 7 + i$$

$$z_2 = 1 - 3i$$

Again this calculation could be carried out on the TI-Nspire:

Comparing Real and Imaginary parts:

An alternative method to answering this question is to consider the real and imaginary parts separately. This will give rise to a system of 4 equations and 4 unknowns. (It is important to add that solving it this way is not a required part of the course, as the IB only require solving systems of 3 equations and 3 unknowns. However it is a valid method based on techniques covered in the IB).

Let:

$$z_1 = a + ib$$
$$z_2 = c + id$$

This means our original equations can be rewritten as:

$$2z_1 + z_2 = 2(a + ib) + c + id = 7 + i \quad (A)$$
$$iz_1 - z_2 = i(a + ib) - (c + id) = -3 + 6i \quad (B)$$

Considering the real components of these gives:

$$2a + c = 7$$
$$-b - c = -3$$

The imaginary components give:

$$2b + d = 1$$
$$a - d = 6$$

Solving this system is not something that you would do by hand in an exam situation, as it would take too long. However the TI-Nspire has a built-in method for solving systems of linear equations which can be accessed by pressing (menu)(3)(2), which will launch the wizard. Change the number of variables to 4, and enter the variable names as a,b,c,d. Then enter the equations we generated above:
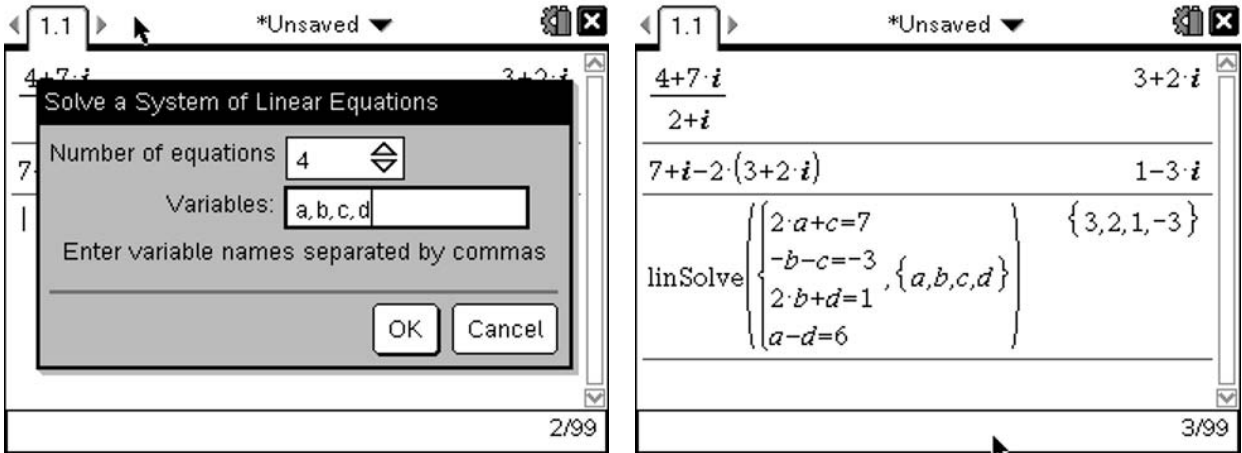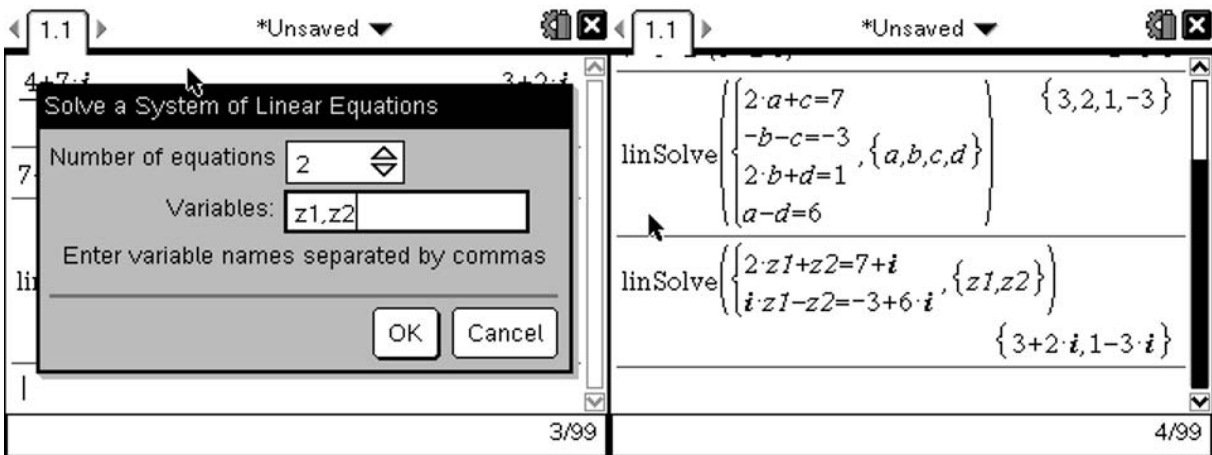

Our solutions are:

$$z_1 = 3 + 2i$$
$$z_2 = 1 - 3i$$

Complex Systems of Equations:

However it is possible to solve this system directly on the TI-Nspire using the same method we used above. The wizard for solving systems of linear equations can be accessed by pressing (menu)(3)(2). This time however, we are going to use just two variables z_1 and z_2.

Which again gives us that:

$$z_1 = 3 + 2i$$
$$z_2 = 1 - 3i$$

Markscheme:

$2z_1 + z_2 = 7 + i \quad (A)$
$iz_1 - z_2 = -3 + 6i \quad (B)$
$2z_1 + iz_1 = 4 + 7i \quad (A) + (B)$ M1

$(2 + i)z_1 = 4 + 7i$ (M1)

$z_1 = \dfrac{4+i}{2+i}$ (A1)

$z_1 = \dfrac{4+7i}{2+i} \times \dfrac{2-i}{2-i}$ (M1)

$z_1 = 3 + 2i$ A1

$z_2 = 1 - 3i$ A1

Whilst it is possible to obtain the answers with little or no working as demonstrated above this approach is likely to mean that you miss out on many of the marks in this type of question. Correct answers here are likely to only gain 2 out of the possible 6 marks, so it is important that you show working to make your method explicit.

However, the inclusion of the first line of the solution, implying the strategy, with correct answers would score full marks.

Poisson Distribution:

> **Question:**
> The random variable X follows a Poisson distribution. Given that $P(X \leq 4) = 0.4$, find:
> i) The mean of the distribution
> ii) $P(X \geq 7)$
>
> [5 Marks]

Given that X follows a Poisson Distribution, we know that it is discrete. Therefore we know that:

$$P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 0.4$$

But we also know that $P(X = x) = \frac{m^x e^{-m}}{x!}$, so we have:

$$\frac{m^0 e^{-m}}{0!} + \frac{m^1 e^{-m}}{1!} + \frac{m^2 e^{-m}}{2!} + \frac{m^3 e^{-m}}{3!} + \frac{m^4 e^{-m}}{4!} = 0.4$$

$$e^{-m} + m e^{-m} + \frac{m^2 e^{-m}}{2} + \frac{m^3 e^{-m}}{6} + \frac{m^4 e^{-m}}{24} = 0.4$$

$$\frac{e^{-m}}{24}(24 + 24m + 12m^2 + 4m^3 + m^4) = 0.4$$

However this equation has no analytic solution, so we have to find a numerical approximate solution. We can do this using the *nSolve* function like this:

$$nSolve\left(\frac{e^{-m}}{24}(24 + 24m + 12m^2 + 4m^3 + m^4) = 0.4, m, 0\right)$$

The format here is the equation, followed by the variable you want to solve for. For this example, include the 0 at the end which acts as the lower bound for the solution.
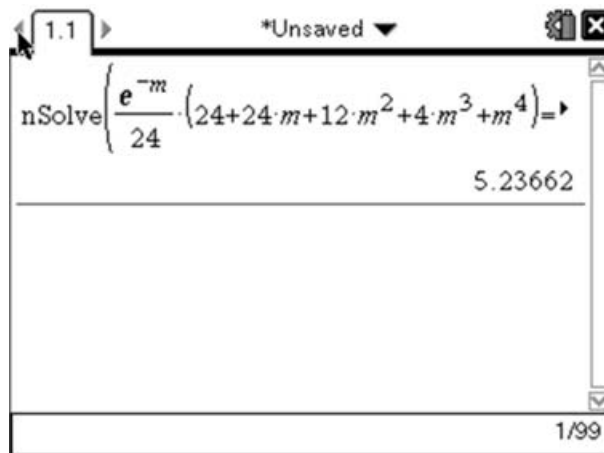

As with all functions on the TI-Nspire, you can either enter the functions by just typing in the function names (i.e. Ⓝ Ⓢ Ⓞ Ⓛ Ⓥ Ⓔ ⓘ etc…) or by using the menu system (menu ③ ①).
However, as the TI-Nspire has a built-in function for the Poisson Distribution, we can use that instead, which gives us a much neater function:

$$nSolve(PoissCdf(m, 0, 4) = 0.4, m, 0)$$

Here the syntax of the PoissCdf function ((menu)(5)(5)(1)) is the **mean**, the **lowerbound**, then the **upperbound**. However as we don't know the value of the mean, we put this in as a variable (in this case m) and then solve for m.
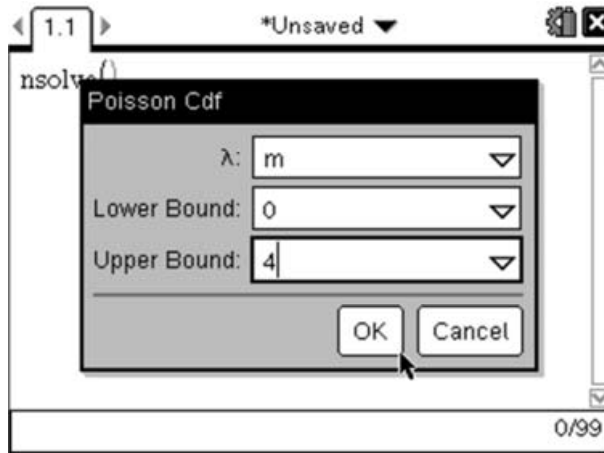
You will probably receive an error message at this point, as when you select OK from the dialogue box it tries to run the function, but we have not yet finished entering the information needed for the nSolve function. Simply select OK and finish entering the remaining information then press (enter).
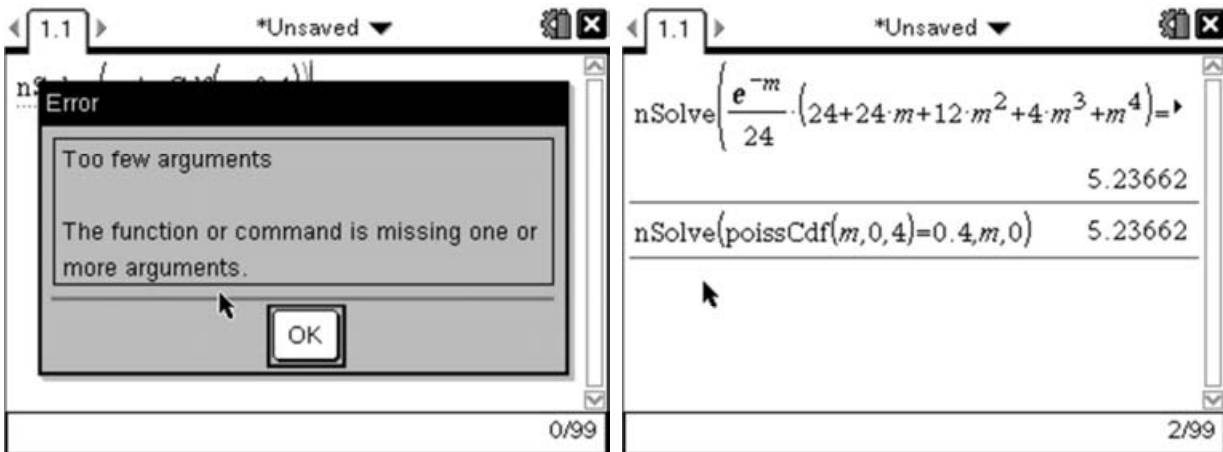
Both of these approaches return the same value for the mean of m=5.2366181156977

Now that we have the value for the mean we can easily work out the value for $P(X \geq 7)$. As the Poisson Distribution has no upperbound, the typical method for approaching this question is to realise that, as the sum of the distribution must be 1, we have:
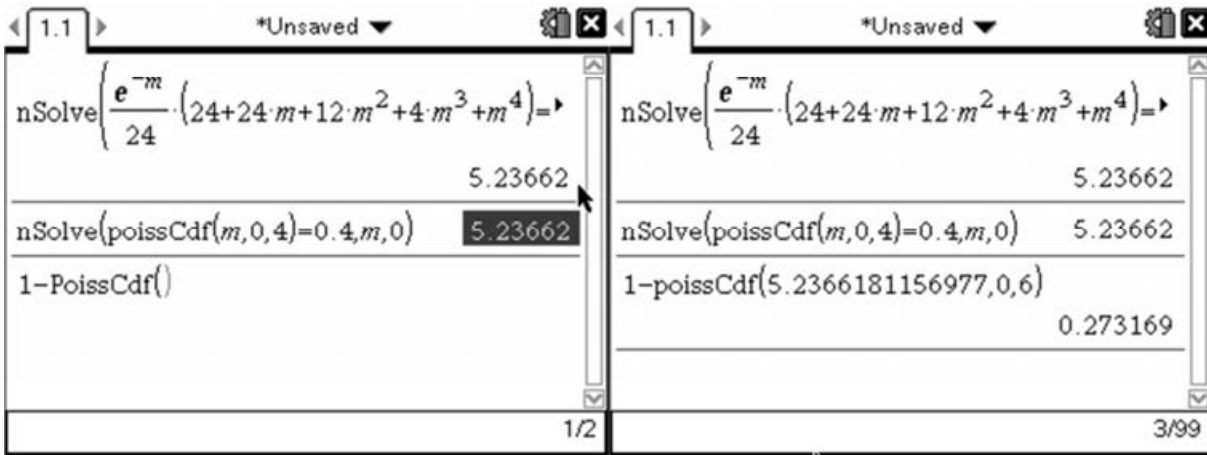
$$P(X \geq 7) = 1 - P(X < 7)$$

But as the Poisson Distribution is discrete this is the same as:

$$P(X \geq 7) = 1 - P(X \leq 6)$$

Using the Poisson Cumulative Distribution function (PoissCdf), we can evaluate this as:

$$1 - PoissCdf\,(5.2366181156977, 0, 6)$$

Instead of using the 'wizard', we can type the PoissCdf function using the letter keys at the bottom of the keypad. Rather than retyping the mean that we calculated earlier, we can reuse the value by pulling it out of the history. This is done by pressing up (▲) until the value is highlighted then pressing the (enter) key to copy it down.

If we want to use the wizard then it would be sensible at this stage to store the calculated value for the mean as a variable. We can do this as follows:

Type 'm'(Ⓜ) then enter the define function which looks like ⌈:=⌉ (press (ctrl)(ⓘ⁙⁙)). Then press up (▲) until the calculated mean is highlighted, then press the (enter) key to copy it down. Press (enter) again to store the value.
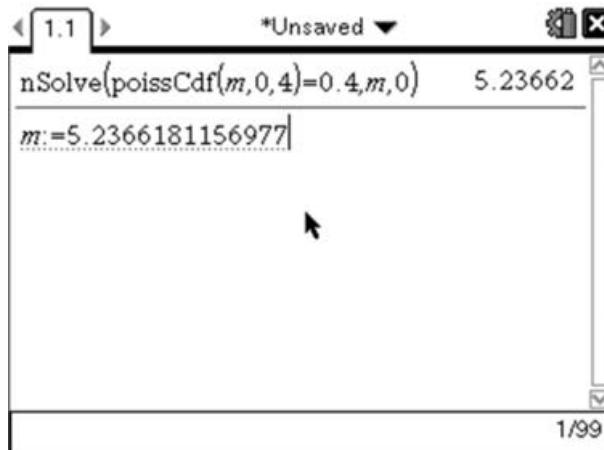
Now we can use this value within the 'wizard' screen. We want '1-PoissCdf(m,0,6)', so we must first type ①⊝ . Then we can use the PoissCdf wizard by pressing ⓜⓔⓝⓤ⑤⑤Ⓘ. Now on the wizard screen we can enter the mean as 'm' as we have stored it as this variable.
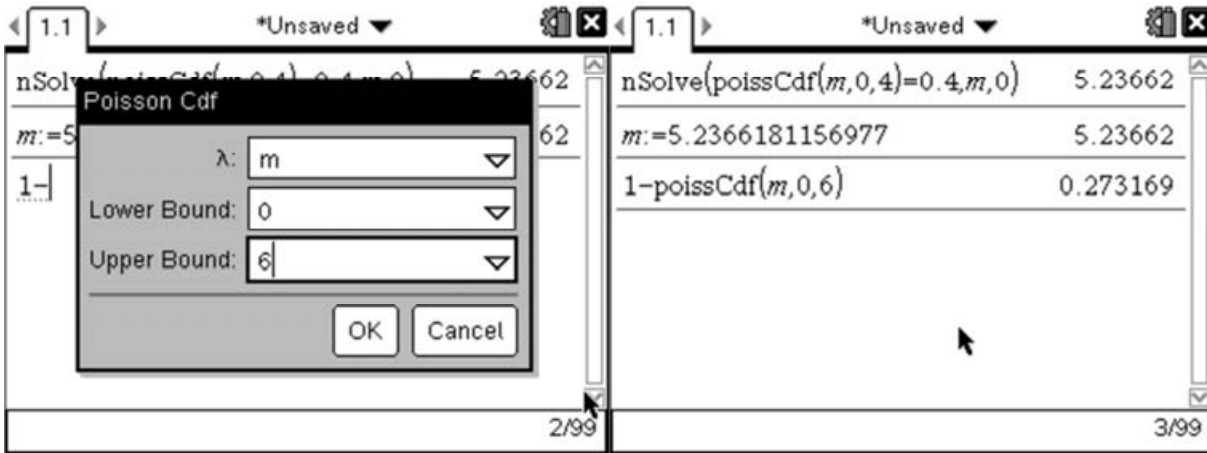
Markscheme:

$X \sim Po(m)$

$$\sum_{x=1}^{4} \frac{m^x e^{-m}}{x!} = 0.4 \qquad\qquad\qquad \text{(M1)}$$

$\Rightarrow m = 5.24 \ (3sf)$ A1

$P(X \geq 7) = 1 - P(X < 7) \quad or \ [1 - P(X \leq 6)]$ M1
$= 1 - 0.7268\cdots$ (A1)
$= 0.273 \ (3sf)$ A1

In this example the key thing again is to make sure the mathematics you carry out is clear and logical. The actual calculations should be carried out by the calculator. So the key things are to state the distribution you are working with and make clear any points where you change the limits due to Poisson or Binomial being discrete.

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